GEORGIA INSTITUTE OF TECHNOLOGY | OFFICE OF CONTRACT ADMINISTRATION

PROJECT ADMINISTRATION DATA SHEET

[x] ORIGINAL [ ] REVISION NO. ______

Project No. G-35-621 GTRI/~~GIT~~ DATE 1-13-83

Project Director: Dr. Anton M. Dainty School/~~Lab~~ Geo Sci

Sponsor: Air Force Office of Scientific Research

Type Agreement: Grant AFOSR-83-0037

Award Period: From 11/15/82 To 11/14/83 (Performance) 1/14/84 (Reports)

Sponsor Amount: Total Estimated: $ 85,000 Funded: $ 36,000

Cost Sharing Amount: $ NA Cost Sharing No: ______

Title: Influence of Scattering on Q In the Lithosphere

ADMINISTRATIVE DATA

OCA Contact Frank H. Huff

1) Sponsor Technical Contact:

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Defense Priority Rating: NA

Military Security Classification: NA
(or) Company/Industrial Proprietary: ______

RESTRICTIONS

See Attached AFOSR Supplemental Information Sheet for Additional Requirements.

Travel: Foreign travel must have prior approval – Contact OCA in each case. Domestic travel requires sponsor approval where total will exceed greater of $500 or 125% of approved proposal budget category.

Equipment: Title vests with GIT for items costing $1,000 or less; however, no acquisitions are proposed in budget. See attached Supplemental Sheet (items c & d). See also Article II.

COMMENTS:

The Government may provide support for a second year at the rate of $49,000.

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G-35-621 closed by termination to G-35-665

GEORGIA INSTITUTE OF TECHNOLOGY OFFICE OF CONTRACT ADMINISTRATION

SPONSORED PROJECT TERMINATION/CLOSEOUT SHEET

Date 1/24/85

Project No. G-35-665 School/Lab Geo. Sci.

Includes Subproject No.(s) N/A

Project Director(s) Dr. Anton M. Dainty GTRI / GIT

Sponsor Air Force Office of Scientific Research

Title "Influence of Scattering on Q in the Lithosphere"

Effective Completion Date: 11/14/84 (Performance) 1/14/85 (Reports)

Grant/Contract Closeout Actions Remaining:

- [ ] None
- [XX] Final Invoice or Final Fiscal Report
- [ ] Closing Documents
- [XX] Final Report of Inventions
- [XXX] Govt. Property Inventory & Related Certificate
- [ ] Classified Material Certificate
- [ ] Other

Continues Project No. G-35-621 Continued by Project No.

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G-35-621

R and D Status Report, December 1982 - February 1983

(a) ARPA Order 4397, AM No. 3

(b) Program Code 3D60

(c) Georgia Tech Research Institute
Atlanta, GA 30332

(d) 15 November 1982

(e) 14 November 1983

(f) $36,000

(g) AFOSR-83-0037

(h) Dr. Anton M. Dainty, (404)894-2860 (Principal Investigator)

(i) Mr. William J. Best, (202)767-4908 (Program Manager)

(j) Influence of Scattering on Q in the Lithosphere

Sponsored by

Advanced Research Projects Agency (DOD)
ARPA Order No. 4397, AM No. 3
Monitored by NP Under Grant No. AFOSR-83-0037

Signed:

Anton M. Dainty
Principal Investigator
School of Geophysical Sciences
Georgia Institute of Technology

(a) Progress

This is the first report under the grant. Three items will be discussed. One is the continuation, with some support from the grant, of work previously supported by Georgia Tech on coda analysis of very close events. A second item refers to theoretical work on scattering in the limit of high and low frequencies from random media. The third item is the training of graduate students.

Work has been carried out for the last nine months at Georgia Tech on analysis of the codas of digital recordings of events at Mammoth Lakes, California, and Monticello, South Carolina, taken by U.S.G.S. While the work has not in the main been supported by this grant, it is extremely relevant. The work has been carried out by a Master's student; reproduction of his thesis was supported (and acknowledged) by this grant, since it can serve as a "training manual" for future graduate assistants. The events, which were small and recorded at close range, were analysed to obtain coda Q (from coda decay) and turbidity g (from analysis of the change in coda Q with frequency, and also from the coda excitation) for the frequency range 3 to 72 Hz. One important and surprising result was that coda Q was similar in the two regions. This does not agree with other workers' results, which indicate that Q in tectonic regions such as Mammoth Lakes is lower at frequencies of 1 to 10 Hz than Q in non-tectonic regions such as Monticello. One explanation for the discrepancy is that in our work the events were closer to the recorders and the codas much shorter (less than 20 seconds total coda length), thus limiting the region sampled by the coda waves to the crust. Then our results indicate that scattering attenuation in the (continental) crust is similar from place to place, and that other workers' results showing differences in Q between tectonic and non-tectonic regions must refer to differences in the upper mantle.

As part of the analysis of these records, an attempt was made to separate the observed coda Q into a contribution due to anelastic attenuation and a contribution due to scattering ("turbidity"). Two methods were used: a method which assumes that all the variation of Q with frequency is due to scattering, and a method that derives the turbidity independently from the coda excitation. The results from these two analyses do not agree. Analysis by coda excitation indicates, in the main, that the effects of scattering are small. Analysis by variation of Q with frequency indicates that the effects of scattering are large (an order of magnitude larger than analysis by Aki's method) and that multiple scattering may be occurring, a conclusion that is supported by other features of the codas. The reason for the difference is currently being investigated.

Another topic that has been studied is the theory of scattering in random media. Results obtained so far indicate that for random velocity fluctuations characterized by an autocorrelation function, scattering tends to Rayleigh scattering (i.e., scattered intensity varies as the fourth power of frequency) for low enough frequencies. A more interesting result demonstrates that for high frequencies backscattered intensity is proportional to the first derivative of the autocorrelation function at zero lag, and is independent of frequency ("geometrical" scattering). However, for any "smooth" velocity fluctuation (i.e., one with no discontinuities) the appropriate derivative is zero, and there is no backscattered energy. Further, the frequency range 1 to 30 Hz appears to be "high frequency" for the purposes of the theory, and the scattering observed implies that there must be discontinuities in the earth causing it.

Finally, a graduate assistant is being trained for data collection and analysis to be carried out starting in the summer.

(b) Major Equipment Purchased - Not applicable.

(c) Changes in Key Personnel - None.

(d) Trips, Meetings, etc. - None.

(e) Problems - None.

(f) Deviations from Planned Effort - None.

(g) Fiscal Status

1.	Funds Currently Supplied:	$36,000
2.	Expenditures and Commitments to Date:	$ 5,247.74
3.	Funds Needed to Complete Work:	$30,752.26
4.	Estimated Completion Date:	14 November 1983

6-35-621

R and D Status Report -- 1 March 1983 - 31 May 1983

(a) ARPA Order 4397, AM No. 3

(b) Program Code: 3D60

(c) Contractor: Georgia Tech Research Institute
Atlanta, GA 30332

(d) Start Date: 15 November 1982

(e) Expiration Date: 14 November 1983
(with option until 14 November 1984)

(f) Amount: $36,000 ($85,000 with option year)

(g) Contract Number: AFOSR-83-0037

(h) Principal Investigator: Anton M. Dainty (404) 894-2860

(i) Program Manager: William J. Best (202) 767-4908

(j) Title: Influence of scattering on Q in the lithosphere

Sponsored by

Advanced Research Projects Agency (DOD)
ARPA Order No. 4397, AM No. 3
Monitored by NP Under Grant No. AFOSR-83-0037

Signed:

Anton M. Dainty
Principal Investigator
School of Geophysical Sciences

(a) Progress

Three areas were investigated during the reporting period: the estimation of S-wave spectra to determine the turbidity, a parameter that describes the strength of scattering; theoretical work on high frequency seismograms; and obtaining of new digital seismograms for eastern North America.

In single scattering theory, the strength of scattering may be described by the turbidity, which is the energy scattered out of a plane wave as it traverses unit distance. We are attempting to measure the turbidity for a series of local earthquakes from the ratio of the coda power to the S-wave power. The earthquakes occurred near Mammoth Lakes, California, and Monticello, South Carolina, and have previously been analyzed to obtain coda Q as a function of frequency. The turbidities obtained are smaller than those implied by the Q. This is contrary to the findings of other authors, but it is in accord with theoretical results discussed in the next paragraph.

Theoretical analysis has continued on high-frequency back-scattering from a random medium. Expressions have been derived for the turbidity and the Q due to scattering. These expressions indicate the turbidity is independent of frequency, provided the medium has discontinuities; if the medium is "smooth", the turbidity tends to zero. The Q due to scattering will only exist if the medium has discontinuities and is proportional to the frequency and the inverse of the turbidity. The results also indicate that the turbidity obtained from the Q should be an order of magnitude larger than the turbidity obtained by the method described in the second paragraph. This is in approximate accord with the results obtained.

We are starting efforts to find additional digital seismograms from the eastern United States. We require seismograms at relatively near distances, preferably around 10 km and certainly no farther than 100 km, with coda lengths ranging from 10 to 100 sec, to enable us to selectively examine scattering in the crust and the mantle. Three approaches are being proposed: The first is to obtain data locally using the Georgia Tech network, probably from blasts--there are at least two appropriate sites in the southeastern U.S. The second approach is to acquire data from aftershocks of the New Brunswick earthquake of January 9, 1982; we intend to approach both U.S.G.S. and the Canadian authorities to see if such data can be obtained. Finally, we may go to an area of natural seismicity, probably the New Madrid area, to record events.

(b) Major Equipment Purchased - Not applicable.

(c) Changes in Key Personnel - None

(d) Meetings: Fifth Annual DARPA/AFOSR Symposium on Seismic Detection Analysis, Discrimination and Yield Estimation, Eastsound, WA, May 16-18. Discussed theory of scattering from random media and possible sites for acquisition of digital data in eastern U.S.

(e) Problems - None

(f) Deviations from Planned Effort - None

(g) Fiscal Status

1. Funds Currently Supplied: $36,000.00

2. Expenditures and Commitments to Date: 8,680.01

3. Funds Needed to Complete Work: 27,319.99 (76,319.99 with option year)

4. Estimated Completion Date: 14 November 1983 (14 November 1984 with option year)

6-35-621

R and D Status Report -- 1 June 1983 - 31 August 1983

(a) ARPA Order 4397, AM No. 3

(b) Program Code: 3D60

(c) Contractor: Georgia Tech Research Institute, Atlanta, GA 30332

(d) Start Date: 15 November 1982

(e) Expiration Date: 14 November 1984

(f) Amount: $85,000

(g) Contract Number: AFOSR-83-0037

(h) Principal Investigator: Anton M. Dainty (404) 894-2860

(i) Program Manager: William J. Best (202) 767-4908

(j) Title: Influence of Scattering on Q in the Lithosphere

Sponsored by

Advanced Research Projects Agency (DOD)
ARPA Order No. 4379, AM No. 3
Monitored by NP under Grant No. AFOSR-83-0037

Signed:

Anton M. Dainty
Principal Investigator
School of Geophysical Sciences

(a). Progress

In the reporting period, detailed work was begun on the comparison of the basic scattering parameter, the turbidity, from the amplitude of the coda relative to the amplitude of the S waves using single scattering theory. This parameter, here called the backscattering turbidity, was estimated from records taken from Mammoth Lakes, California, and used previously to estimate the Q. From the Q another turbidity may be calculated assuming all the attenuation is due to single scattering. Theoretically, for high frequency scattering from a "rough" medium (i.e., one with discontinuities), the ratio of the total turbidity to the backscattering turbidity is about 200, constant with frequency. The ratio should never be less than 1.

The results obtained at Mammoth Lakes show that the ratio described above is 0.27 at 3 Hz (less than 1), rises steadily to about 400 at 24-36 Hz (the same order of magnitude as the theory), and then falls to 60 for 72 Hz. The results at the frequency range 3-12 Hz indicate that the assumptions must be wrong, most likely the single scattering assumption. This indicates there may be "smooth" fluctuations in seismic velocity with a scale length of 100 m at Mammoth Lakes. The results at 72 Hz indicate a possible intrusive Q of 3000.

The variation of the scattering parameter with depth of focus was checked at Mammoth Lakes: no significant variation was found.

Efforts continued to obtain seismograms for further study.

(b) Major Equipment Purchases - Not applicable

(c) Changes in Key Personnel - None

(d) Meetings: American Geophysical Union Annual Meeting
Baltimore, Maryland, May 30 - June 3, 1983
Presentation of talk; discussed acquisition of digital data.

(e) Problems - None

(f) Deviations from Planned Effort - None

(g) Fiscal Status

1. Funds currently supplied	$36,000.00
2. Expenditures and Commitments to Date	$20,801.34
3. Funds Needed to Complete Work	$64,198.66
4. Estimated Completion Date	14 November 1984

G-35-621

R and D Status Report -- 1 September 1983 - 14 November 1983

(a) ARPA Order 4397, AM No. 3

(b) Program Code: 3D60

(c) Contractor: Georgia Tech Research Institute, Atlanta, GA 30332

(d) Start Date: 15 November 1982

(e) Expiration Date: 14 November 1984

(f) Amount: $85,000

(g) Contract Number: AFOSR-83-0037

(h) Principal Investigator: Anton M. Dainty (404) 894-2860

(i) Program Manager: William J. Best (202) 767-4908

(j) Title: Influence of Scattering on Q in the Lithosphere

Sponsored by

Advanced Research Projects Agency (DOD)
ARPA Order No. 4397, AM No. 3
Monitored by NP under Grant No. AFOSR-83-0037

Signed:

Anton M. Dainty
Principal Investigator
School of Geophysical Sciences

(a) Progress

In this reporting period, we were able to obtain from the Canadian authorities additional digital seismograms from the Eastern Canada Network. These seismograms are several minutes long and are taken at a distance of 25 km from the epicentral region of the Miramichi, New Brunswick (Canada), earthquake of January 9, 1982. Since the sampling interval is 60 samples/second, analysis will only be possible for frequencies up to about 20 Hz, but the longer codas should enable us to look at deeper regions of the lithosphere. We are also attempting to acquire a closer-in set of shorter codas for this region. During this reporting period, we have been writing appropriate programs to read, plot, and edit the new data set.

(b) Major Equipment Purchases: Not applicable

(c) Changes in Key Personnel: None

(d) Meetings: None

(e) Problems: None

(f) Deviations from Planned Effort: None

(g) Fiscal Status:

1. Funds Currently Supplied	$85,000.00
2. Expenditures and Commitments to Date	$35,704.84
3. Funds Needed to Complete Work	$49,000.00
4. Estimated Completion Date	14 November 1984

G-35-621

Research Progress and Forecast Report

Project Title: "Influence of Scattering on Q in the Lithosphere"

Sponsored by

Advanced Research Projects Agency (DOD)
ARPA Order No. 4397, Am No. 3
Monitored by NP Under Grant No. AFOSR-83-0037

Program Code: 3D60

Contractor: Georgia Tech Research Institute
Atlanta, GA 30332

Starting Date: 15 November 1982

Contract Amount: $36,000 ($85,000 with option year)

Expiration Date: 14 November 1983
(with option until 14 November 1984)

Program Manager: William J. Best (202) 767-4908

Anton M. Dainty
Principal Investigator
School of Geophysical Sciences
(404) 894-2860

Three areas of work have been pursued under this contract. One is the acquisition of data, specifically digital data consisting of waveforms of codas from local earthquakes in the eastern United States. Another area is the development of programs to interpret such data; a beginning has been made on the interpretation of data. Finally, a theoretical study has been completed to assist the interpretation.

In the area of data acquisition, the type of data required is digital data, preferably with a sample rate of at least 100 samples/sec. "Local" events, i.e., within 100 km of the seismometer, are needed with a frequency range of 0.1 to 30 Hz, if possible. The object of the study is the coda, which may last up to two or three minutes after origin time, depending on the size of the event. Two approaches are being followed to acquire this data--locating data taken by other organizations and collecting data with our own instruments. Suitable data for shorter codas (up to 30 seconds after origin time) has been supplied by the U.S. Geological Survey. We have asked both the U.S.G.S. and the Department of Energy, Mines and Resources of Canada for data from aftershocks of the New Brunswick earthquake of January 9, 1982--so far, the Canadians have agreed. Using our own instruments, we have obtained suitable recordings of blasts in the southeastern United States and may also attempt to gather data in Arkansas, where there is an ongoing earthquake swarm.

A complete set of programs has been written to interpret digital coda data by standard methods, originally due to Aki and Chouet

(1975). Two parameters are determined: the coda decay and the ratio of coda amplitude (corrected for attenuation) to the direct S-wave amplitude (also corrected for attenuation) as a function of frequency. Both measurements may be expressed in terms of a turbidity, which is the fraction of energy scattered per unit distance travelled by a wave. The turbidity associated with the decay, known as the total turbidity, refers to all scattered energy that is lost from the wave. The rate of decay of energy in a plane wave may then be expressed as

$$E = E_0 \exp[-g_t x] \tag{1}$$

where E is the energy at position x, E_0 is the energy at x = 0, and g_t is the total turbidity. In terms of Q due to scattering, Q_s,

$$\frac{1}{Q_s} = \frac{g_t v}{2\pi f} \tag{2}$$

where v is the wave velocity and f is frequency. Since in general there is also attenuation due to imperfect elasticity, the turbidity g_{app} measured by this method will be greater than g_t according to the relation

$$g_{app} = g_t + \frac{2\pi f}{vQ_i} \tag{3}$$

where Q_i is the Q due to imperfect elasticity. Preliminary results in two areas indicate that the second term in (3) is small in the crust for f > 1 Hz, attenuation in this region is mainly controlled by scattering, g_t is approximately independent of frequency and has a value of about 0.01 km^{-1}.

The turbidity measured from the ratio of the coda energy to the energy of the direct S wave refers to energy scattered back towards the seismometer and is known as the backscattering turbidity, g_b. It is proportional to the ratio just discussed. From the definition of the various types of turbidity,

$$g_{app} \geq g_t \geq g_b \tag{4}$$

Analysis of g_b is just commencing.

Two questions have been examined in the theoretical studies--the frequency dependence expected for the total and backscattering turbidity and the relationship between them. For a medium with "scatterers" of some characteristic dimension, the turbidities are independent of frequency for wavelengths shorter than the characteristic dimension of the scatterers. This appears to agree with results obtained so far. The relationship between the turbidities is of interest because it could enable an independent determination of g_t from g_b leading to a comparison with g_{app} via (3). Theoretical results indicate that g_b should be between 1 and 10% of g_t.

I do not anticipate any difficulty obtaining data or running programs. The principal potential difficulty is a lack of theoretical understanding of scattering, especially multiple scattering. Analysis has suggested that multiple scattering may be occurring in the coda. Elucidation of this problem, or even an unequivocal indication that multiple scattering is occurring, would be a considerable advance.

Reference:

Aki, K., and Chouet, B. (1975). Origin of coda waves: source, attenuation, and scattering effects. J. Geophys. Res., 80, 3322.

G-35-621

Semi-Annual Technical Report No. 1

1 December 1982 - 31 May 1983

"Influence of Scattering on Q in the Lithosphere"

By

Anton M. Dainty and Robert M. Duckworth

School of Geophysical Sciences
Georgia Institute of Technology
Atlanta, Georgia 30332

Principal Investigator: Anton M. Dainty (404) 894-2860

Contract Amount: $36,000 ($85,000 with option year)

Contract Termination Date: 14 November 1983
(14 November 1984 option)

Program Manager: William J. Best (202) 767-4908

Sponsored by

Advanced Research Projects Agency (DOD)

ARPA Order No. 4397, Am No. 3

Monitored by NP Under Grant No. AFOSR-83-0037

Program Code: 3D60

TABLE OF CONTENTS

TECHNICAL REPORT SUMMARY

This project studies the influence of scattering on the attenuation of seismic waves, parameterised in terms of the quality factor Q, in the lithosphere. An understanding of attenuation mechanisms in the lithosphere is vital if accurate yield estimates are to be made from regional seismic phases, which travel in this region of the earth. An understanding of scattering in the lithosphere could lead to a greater understanding of the causes of variability in yield estimates, since such scattering is suspected to be the cause of at least some of this variability.

The first part of this report examines efforts to measure Q as a function of frequency from the coda of seismograms taken from Mammoth Lakes, California, and Monticello, South Carolina, and to estimate backscattering from the excitation of the coda relative to the direct shear wave. The coda consists of late arriving, apparently scattered, energy. We have used Aki's theory, which considers the coda to be singly backscattered waves, to interpret our results. Measurement of Q from the decay of the coda with time at the two sites has led to the following conclusions:

1. The apparent Q increases (attenuation decreases) with the length (in time) of coda examined for frequencies of 3-72 Hz at both sites. The effect is particularly dramatic at Monticello, where the coda decay changes at about 10 sec after origin time. This indicates either a change in scattering from single scattering to multiple scattering at this time, or alternatively an increase in Q with depth.

This increase would occur at 15 to 20 km depth under Monticello.

2. The value of Q in the frequency range 3-72 Hz is approximately the same at both sites, especially for codas less than 10 seconds long. If this holds for other sites, it may indicate that Q in the (upper) crust is similar from place to place, and that differences in Q from region to region are due to differences in the lower crust or upper mantle.

3. At both sites, Q is approximately proportional to frequency. This suggests that high frequency geometrical scattering is the dominant attenuation mechanism. The scattering may be characterized by the turbidity, which is the fractional loss of energy due to scattering per unit distance travelled by a plane wave. Values of this parameter are between 0.1 and 0.01 km^{-1}, depending on the portion of the coda and the assumptions made.

4. The backscattering turbidity is the fraction of energy per unit travel distance scattered in the direction opposite to the wave propagation direction. This parameter may be estimated from the relative excitation of the coda compared to the direct S wave. Values obtained are in the range 0.02-0.003 km^{-1}, i.e., between one and two orders of magnitude less than the turbidity discussed in (3) above. This is in accord with the theoretical results given in the second part of the report.

The second part of the report deals with a theoretical investigation into high frequency acoustic backscattering. The starting point for the investigation is a formula derived previously by Pekeris and Chernov for the scattered intensity from a region containing random fluctuations of velocity characterized by an autocorrelation function. This integral expression is evaluated by expanding the autocorrelation in a Taylor series about zero lag. The resulting formula is valid for high frequencies and backscatter or sidescatter. Conclusions are:

1. The scattered intensity is independent of frequency and proportional to the first derivative of the autocorrelation at zero lag. The lack of frequency dependence is analogous to high frequency geometrical scattering from an obstacle.
2. For the autocorrelation to have a non-zero first derivative at zero lag, the velocity fluctuations cannot be "smooth", i.e., either the velocity or its first derivative must be discontinuous. Smooth velocity fluctuations do not produce backscattered energy at high frequencies.
3. The turbidity involved in attenuation of plane waves is between 10 and 100 times greater than the backscattering turbidity, depending on the amount of scattered energy that is considered lost.

OBSERVATIONS OF CODA Q FOR THE CRUST NEAR MAMMOTH LAKES, CALIFORNIA, AND MONTICELLO, SOUTH CAROLINA

By Robert M. Duckworth and Anton M. Dainty

Introduction

The seismic Q of a material is a measure of the degree to which it attenuates the harmonic energy content of seismic waves passing through it. Seismic energy released by earthquakes and underground explosions propagates in the earth primarily in the form of elastic waves. To the extent that earth materials are not perfectly elastic the elastic waves will be attenuated. The energy at an initial time, E_0, and the energy E after elapsed time t are related to the Q by

$$E = E_0 \exp(-2\pi ft/Q) \qquad (1)$$

where f is the frequency of the wave. Attenuation is proportional to the distance a wave travels, x, since t = x/B where B is the speed at which the wave propagates. Therefore, knowledge of Q should be useful in predicting the expected amplitudes of seismic waves for yield estimation and other applications. However, because earth materials are inhomogeneous a prediction of seismic wave amplitude is complicated.

Inhomogeneities lead to reflections and refractions, or scattering, of seismic waves. Scattering by these inhomogeneities is an important part of the seismic trace recorded as a function of time. The first part of a seismogram of a local (within a few hundred km of the seismic recording station) earthquake is the Primary or P-wave. This is a compressional wave and travels at speeds up to about 8 km/s for local earthquakes. A second major feature is the Secondary or S-wave which is a shear wave and travels at speeds of about P-speed/$\sqrt{3}$. Between the P and S arrivals and following the S arrival additional waves arrive. Their amplitude decays in a generally exponential manner. Aki (1969) proposed that these waves are the result of scattering of surface waves by surface inhomogeneities. He gave them the name Coda [Latin: tail] waves. Later, Aki and Chouet (1975) considered body wave scattering. Also, scattering affects the attenuation of seismic waves in two ways. Energy is scattered out of the primary wave decreasing the apparent Q. Some of this scattered energy arrives at the receiver in the form of coda waves. The amplitude of these depends upon the scattering mechanism, characterised by the turbidity as described below, and the inelastic Q.

This report presents estimates of the apparent Q from the coda of local earthquakes in two different geologic regions. Attempts are made to separate the effects of scattering and inelasticity in order to determine the turbidity and inelastic Q for each region. The excitation of the coda has also been considered.

Theory

Coda waves are generally considered to be backscattered energy. A model from Aki and Chouet (1975) of coda waves as single, S-wave to

S-wave scatter from a random distribution of scatterers is used in this report.

Observations show the coda to be independent of source-receiver distance, R, for local coda waves (Aki, 1969; Aki and Chouet, 1975; Rautian and Khalturin, 1978). Sato (1977) lends support to this observation with a model for single isotropic scatter. He shows the time dependence of the mean energy density of the coda near the source to be nearly independent of source-receiver distance R for $R \ll Bt/2$. B is the velocity of S-waves in the medium and t is their travel time measured from the origin time. Because the coda shape is nearly independent of source receiver distance for local coda waves, the model is simplified to the case of coincident source and receiver.

Knopoff and Hudson (1967) modeled scatter from small perturbations in an elastic medium. They found the amplitude for S to S scatter to dominate that of P to P scatter by $(\alpha/B)^2$ for geometric (high frequency) and $(\alpha/B)^4$ for Rayleigh (low frequency) scatter and P to S and S to P scatter to be "vanishingly small" for geometric scatter. α and B represent the speeds of P and S waves respectively. For the crust $\alpha = \sqrt{3}\, B$ is a good approximation and the amplitude of S to S scatter is greater than P to P scatter by a factor of 3 for geometric scatter. The observed character of coda waves also supports S to S dominance (Aki, 1980).

Before presenting a model of coda waves the effects of attenuation will be considered. Plane waves traveling in a medium with a random distribution of scatterers will be attenuated. Let E_0 represent the average energy of the waves before scattering and E represent the energy of plane waves traveling in the same direction as the primary waves after scattering. We assume the primary cause of attenuation by scattering will be backscatter. For $(E_0-E) << E_0$ over a distance equal to the average separation of "backscatterers", the attenuation of primary waves is given by

$$E/E_0 = \exp(-x\rho\sigma_b). \qquad (2)$$

The backscatter cross section σ_b represents the average of the product of the backscatter function and the projected area of the scatterers, or the average apparent area of the scatterers. ρ is the number of scatterers per unit volume. As σ_b and ρ are unknown for the crust, their product is often replaced by a scattering coefficient, g, called the turbidity. The turbidity represents the fractional loss of energy due to scatter per unit distance traveled. The term was introduced by Chernov (1960) and first used in seismic literature by Galkin et al. (1970). They employ it to describe their observed high frequency spatial variation of seismic amplitudes as a function of distance from the source. Equation (2) may also be applied to the attenuation a plane wave suffers when passing through a medium where scattering is occurring as a result of random fluctuations in the medium properties rather than discrete "backscatterers". In this case, g is a function of the statistical properties of the medium and the wavelength (Wu, 1982).

In the earth medium there will also be attenuation resulting from inelasticity. This is attenuation that occurs as elastic energy is converted to heat. For $(E_0-E) << E_0$ over a wavelength, the inelastic attenuation is given by

$$E/E_0 = \exp(-2\pi fx/BQ). \tag{3}$$

Q is the quality factor, B is the speed of shear waves, x is the distance they travel in the medium and f is frequency. The inelastic attenuation mechanism is not well understood. It is generally observed that Q is independent of frequency over the range of seismic frequencies studied. This is in agreement with the results of Knopoff (1964) who studied Q in the laboratory and found it independent of frequency for dry materials. These two attenuation effects can be combined. In terms of the time the energy spends in the medium $t = x/B$ and for constant B,

$$E/E_0 = \exp(-2\pi ft/Q) \,.\, \exp(-Btg) \tag{4a}$$

$$= \exp[-t(2\pi f/Q_T)]. \tag{4b}$$

Q_T represents the combined effects of scattering and inelastic attenuation.

$$1/Q_T = 1/Q_i + gB/2\pi f \;, \tag{5}$$

where Q_i is the inelastic Q (Dainty, 1981). Dainty and Toksoz (1981) suggest that for a single scatter theory to apply $2\pi f/Q_i \gg Bg$ is a necessary condition, as otherwise there would be sufficient energy for higher order scattering.

The power spectrum of the coda as a function of time under the conditions discussed above has been derived by Aki and Chouet (1975). If the spectrum of the source is $\phi(f|r_0)$ at a reference distance r_0

from the source, the coda power spectrum is

$$P(f|t) = st^{-2m} \exp(-2\pi ft/Q_T) \tag{6}$$

where

$$s = |\phi(f|r_o)|^2 \; 4m\pi r_o^{2m}(B/2)^{1-2m} \cdot g \tag{7}$$

s represents the source term, the turbidity $g = \rho\sigma_b$, and $m = 0.5$ for cylindrical (surface wave) or $m = 1.0$ for spherical (body wave) spreading. The source term, s, will in general depend on the scatterers through its dependence on g. The time t is measured from the origin time of the event.

The average coda amplitude at a particular frequency and time $A(f|t)$ is related to the power spectrum.

$$A(f|t) = [P(f|t)\Delta f]^{1/2} \tag{8}$$

Δf is the band width considered about f. Q_T can therefore be measured from the coda wave amplitude attenuation using equation (6). Sato (1977) points out the single scatter model is only applicable for a coda duration $t < 1/gB$. Longer times imply a significant contribution of higher order scattering which effects the return of scattered energy to the later parts of the coda. The decomposition of Q_T to find Q_i and g depends upon assumptions about their frequency dependence. This and the applicability of the single scatter model will be considered in the section on results.

Data

Two sets of earthquake data are available for analysis courtesy of Paul Spudich (U.S. Geological Survey at Menlo Park, California). Both sets contain digital records of earthquakes and their locations. Program Hypoinverse by Klein (1978) is the method of location. Locations and primary S-wave travel times of the earthquakes selected for analysis have been read from the U.S.G.S. tapes (Tables 1 and 2). The first set is from a series of events associated with the filling of Monticello Reservoir in South Carolina. It contains records of 32 earthquakes, recorded in May and June of 1979 and used in a study of stress drops by Fletcher (1982). He reports they are all magnitude $M < 1.7$. Secor et al. (1982) report the earthquakes as occurring in a heterogeneous quartz monzonite pluton of Carboniferous age. Zoback and Hickman (1982) suggest that the earthquakes are shallow, occurring along existing fractures, and the result of an increase in pore pressure caused by increasing the head at the reservoir. Figure 1 adapted from Talwani and Hutchenson (1982) shows the location of the seismic stations and the reservoirs. Epicenters for the earthquakes considered have been indicated on this figure. They are all within 8 km of the recording stations. Secor et al. (1982) report on the two reservoirs. Parr Shoals reservoir was impounded in 1914 and increased in 1976 to a volume of 0.039 km^3 with a surface area of 18 km^2 and a maximum depth of 11 meters. Monticello reservoir has a volume of 0.5 km^3, a surface area of 27 km^2, and a maximum depth of 48 meters.

Fletcher (1982) describes the instrumentation used to record the Monticello earthquakes. A summary is given here. Sprengnether

Table 1. Locations of earthquakes and recording stations at Monticello, South Carolina.

EVENT/STATION	"S" TIME	RANGE	LOCATION
1271000			34 20.03 N 081 19.62 W
DUC	0.68	2.2	34 20.07 N 081 21.06 W
DON	1.05	3.5	34 21.42 N 081 21.20 W
LKS	0.89	3.0	34 19.95 N 081 17.69 W
1320218			NOT POSSIBLE
JAB	1.66	5.8	34 22.28 N 081 19.47 W
1281119			34 20.70 N 081 20.81 W
JAB	1.31	4.6	
1302328			34 20.49 N 081 20.65 W
SNK	0.56	1.8	34 20.29 N 081 19.54 W
JAB	1.19	3.8	
LKS	1.38	4.7	
1310603			34 18.50 N 081 20.50 W
SNK	1.05	3.7	
LKS	1.48	5.2	
JAB	2.00	7.0	
1281831			NOT POSSIBLE
JAB	1.02	3.57	

Table 2. Locations of Earthquakes and Recording Stations at Mammoth Lakes, California.

EVENT/STATION	"S" TIME	RANGE	LOCATION
1571941			37 32.91 N 118 52.53 W
MGE	3.17	8.0	37 33.67 N 118 47.22 W
HCF	4.37	10.6	37 38.51 N 118 50.98 W
CBR	5.50	15.2	37 40.75 N 118 49.51 W
TWL	4.84	13.7	37 36.93 N 119 00.37 W
ROC	5.67	14.9	37 29.78 N 118 43.16 W
TOM	6.11	18.0	37 33.05 N 118 40.32 W
LKM	5.97	17.3	37 41.80 N 118 56.13 W
1592317			37 37.50 N 118 52.52 W
HCF	2.38	3.0	
FIS	2.58	4.2	37 36.84 N 118 49.82 W
CBR	3.60	7.5	
LKM	3.73	9.6	
MGE	4.03	10.6	
TWL	4.52	11.6	
LAK	5.30	13.1	37 38.49 N 118 43.70 W
TOM	6.78	19.7	
ROC	6.71	19.8	

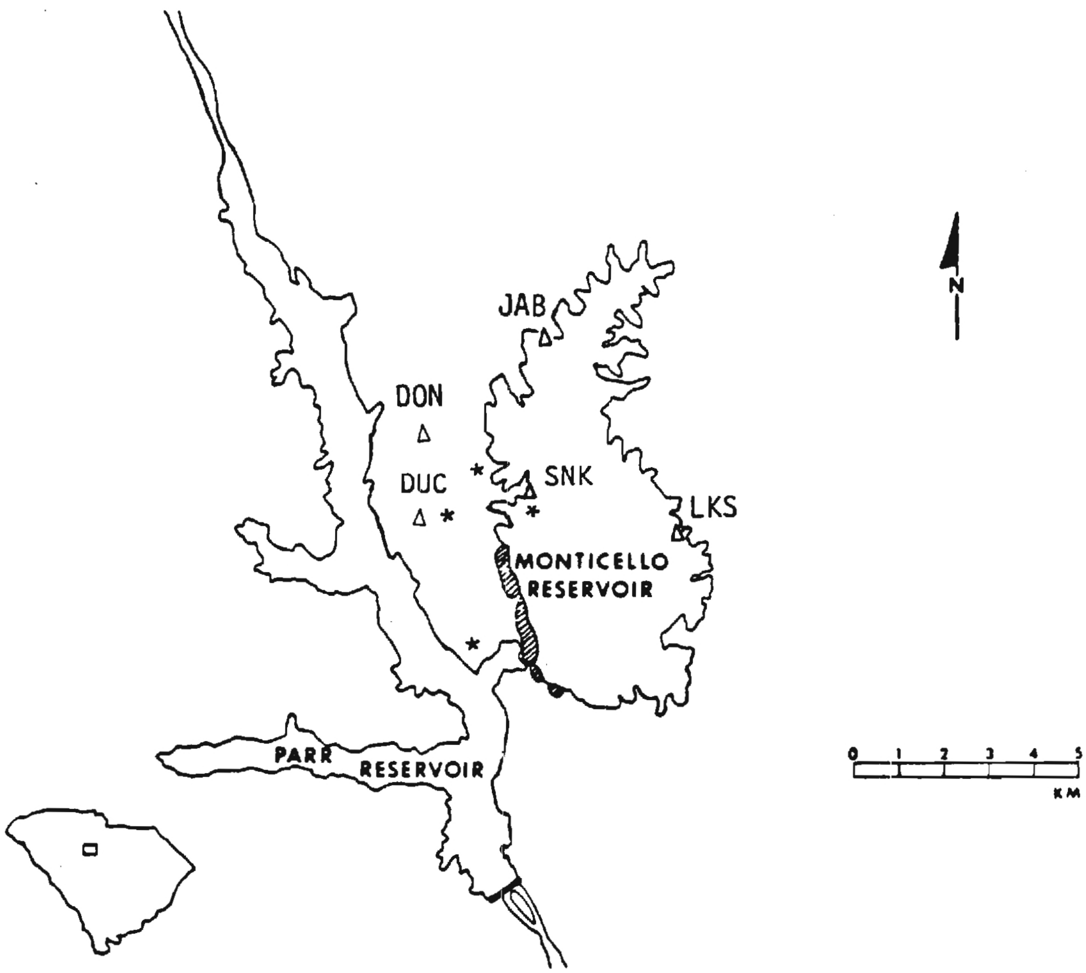


Figure 1. Recording Stations and Epicenters at Monticello Reservoir, South Carolina. stations(Δ), epicenters(*)

DR-100, 12-bit, digital recorders were used with an unspecified make of geophone. The sample rate was 200 Hz with an antialiasing filter of either 50 Hz or 70 Hz having a 30 db per octave rolloff. The geophones were velocity sensors with damping specified as 0.7 of critical. The possibility of aliasing on Monticello records for which the filter corner frequency was set at 70 Hz was considered. Inspection of bandpass filtered records at the higher frequencies revealed no difference in the rate of amplitude decay between the recordings made with a 50 Hz corner filter and a 70 Hz corner filter.

The second set of data is from Mammoth Lakes, California. It consists of the 150 earthquakes used by Archuleta et al. (1982) in a study of source parameters. The earthquakes followed a series of magnitude M > 6.0 earthquakes which occurred May 25, 1980 (Spudich et al., 1981). Wallace et al. (1982) investigate a discrepancy in fault plane projections as reported in the literature for the Mammoth Lakes earthquakes. Citing evidence of an active magma body, they suggest a low velocity zone related to recent magmatic activity as a possible cause. Figure 2 adapted from Spudich et al. (1981) shows the station locations. Epicenters for the earthquakes considered have been added to this figure. The source receiver distances range from 5 to 25 km. The Long Valley Caldera, a volcanic crater 0.7 million years old, is outlined along with major faults. The instrumentation at Mammoth Lakes is essentially the same as at Monticello, except that accelerometers are used at some of the Mammoth stations. Details of the instrumental operations and clock corrections can be found in Spudich et al. (1981).

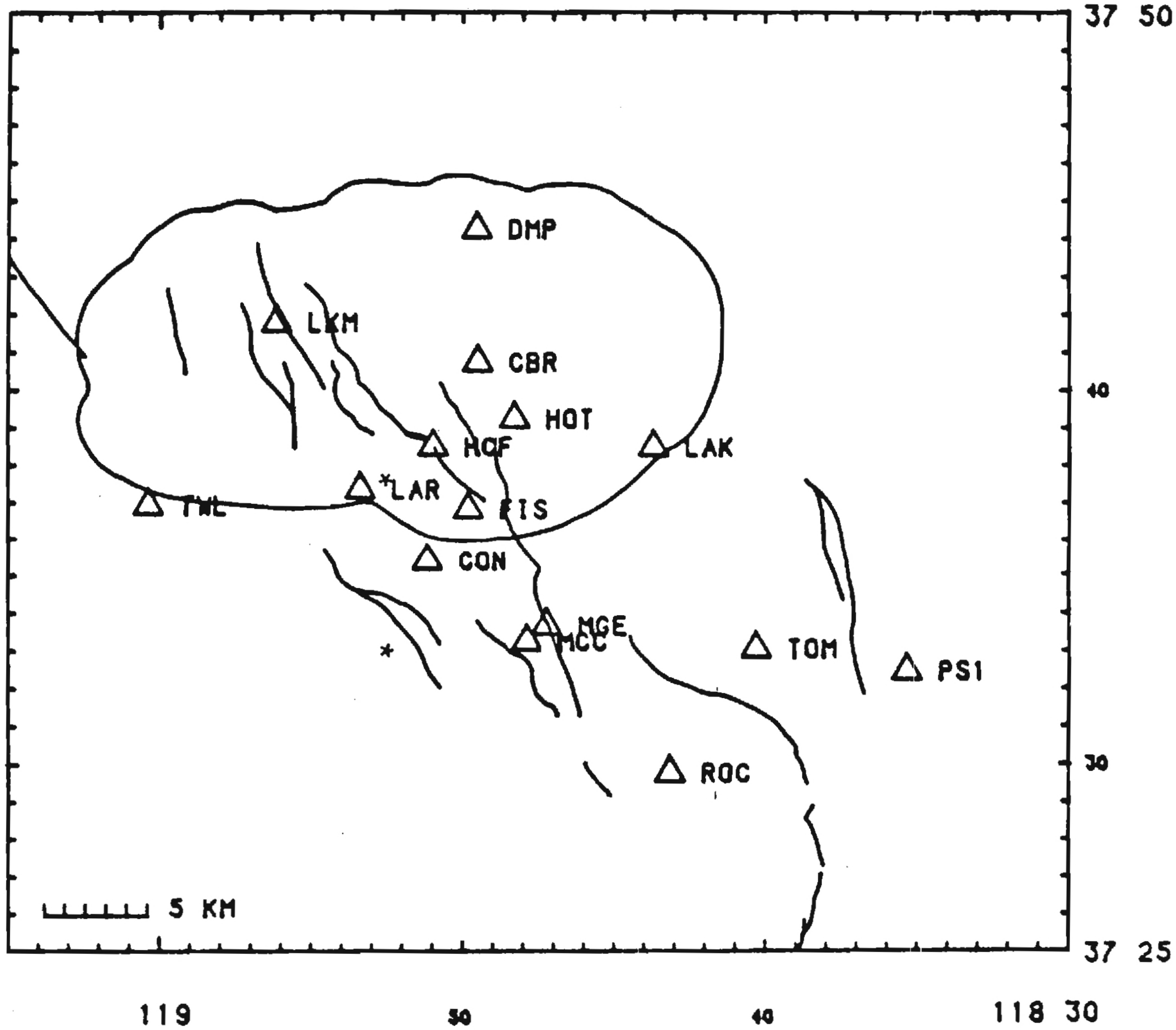


Figure 2. Recording Stations and Epicenters at Mammoth Lakes, California. stations(Δ), epicenters(*)

Analysis

The rms coda amplitude is related to the power spectrum by $A(f|t) = [P(f|t)\Delta f]^{1/2}$. Substituting the equation for the power spectrum in the single scatter model gives

$$A(f|t) = ct^{-m} \exp(-\pi ft/Q_T) \ . \qquad (9)$$

Over the range of frequencies for which geometric scatter is the case, σ_b is constant. Therefore, $c = (s\Delta f)^{1/2}$ has the same frequency dependence as the source for octave Δf. For cylindrical waves $m = 0.5$, while for spherical waves $m = 1.0$. Taking the ordinary logarithm of this expression

$$\log[A(f|t)] = C - m \ . \ \log(t) - bt \qquad (10)$$

is obtained where $b = \log(e)\pi f/Q_T$ and $C = \log(c)$. To estimate Q_T this equation is fit to filtered records by the method of least squares. This is the approach taken by Aki and Chouet (1975).

To study the temporal decay of coda waves the longest records are chosen. Of these the duration of recording ranges from 8.0 to 23.0 seconds at Monticello and from 10.0 to 35.0 seconds at Mammoth Lakes. Only a few records from Monticello are considered to be of sufficient duration for coda analysis (Table 1). The Mammoth Lakes records chosen for analysis are listed in Table 2.

Travel times are taken from the location data provided with the U.S.G.S. tapes. Some of the Monticello earthquakes were not located. For these the S-wave travel time is calculated from the difference in S and P wave arrival times and assumed S and P wave velocities of

3.5 km/s and 6.0 km/s respectively. Ground displacement amplitude as a function of time in octave wide bands is determined for the selected records by the following procedure.

The mean is removed from a record in a 256 sample (1.28 second) window centered about the origin time. This portion of the trace is multiplied by normalized Hamming coefficients. The windowed trace is corrected for instrument gain, Fourier transformed, and multiplied by $1/f$ or $1/f^2$. Assuming a flat instrument response in the frequency band of interest, this converts the trace to ground displacement amplitude from velocity and acceleration records respectively. It has been assumed, as in Archuleta et al. (1982) and Fletcher (1982), that the velocity response is flat from 2 Hz to 50 Hz and the acceleration response is flat above 0.1 Hz. The average amplitude is calculated within octaves (equal power bands) about selected center frequencies. The log of this amplitude is plotted at time zero on separate plots for each frequency. The window is then moved 100 samples (0.5 seconds) and the process is repeated with the log average amplitude plotted at time 0.5. This is done until the end of the record is reached. The result is a series of plots of log average amplitude versus time, measured from origin time, at selected center frequencies from 3 Hz to 72 Hz. Figure 3 contains a sample record and its corresponding filtered amplitude plots. Frequencies above the corner of the antialiasing filter are considered as the constant reduction in amplitude will only show up in the source term. Therefore Q_T can still be estimated provided there is sufficient energy present at these frequencies.

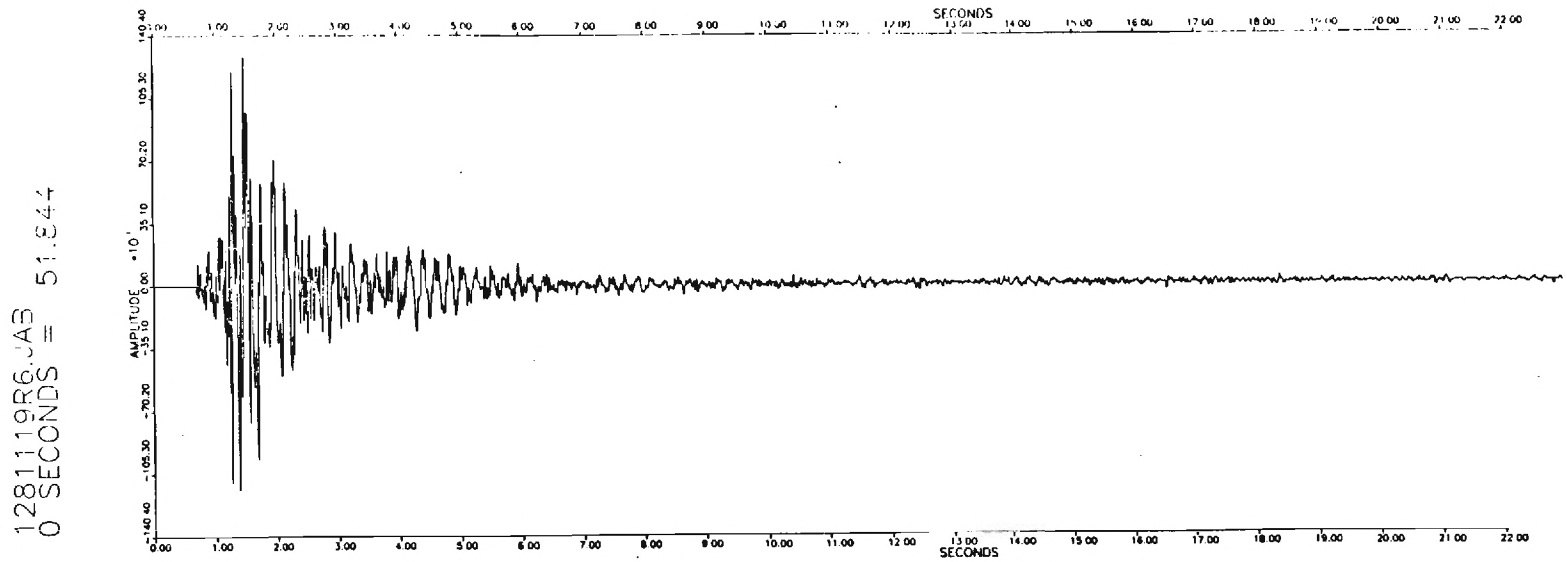
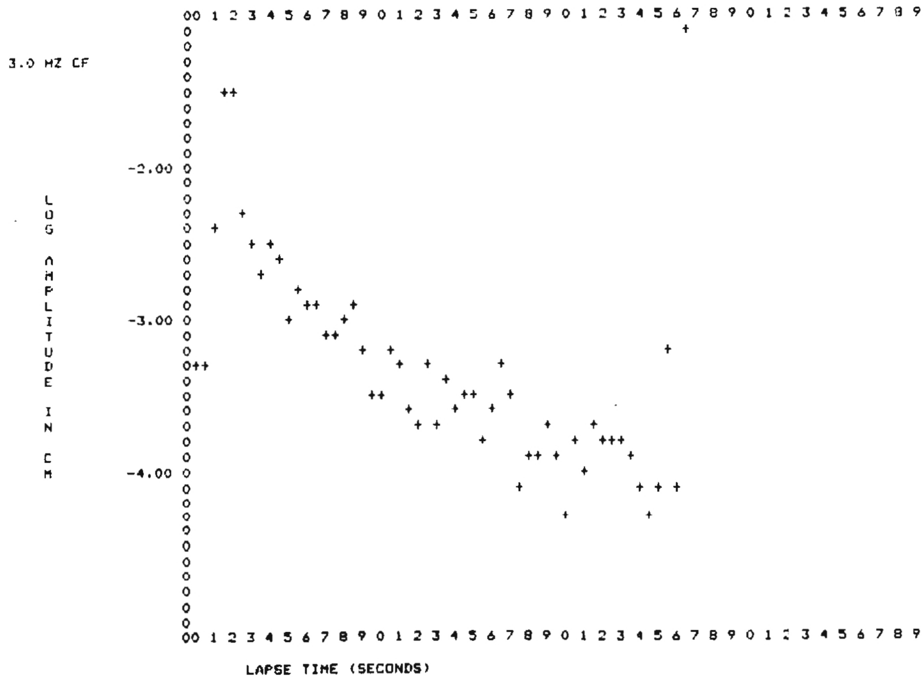


Figure 3a. Record of Event 1281119 at Monticello Reservoir. The amplitude scale is digitizing units.

Figures 3b-h. Log(amplitude) versus Time for Event 1281119at Monticello. (following 4 pages) Plots b-j correspond to center frequencies of 3, 6, 9, 12, 18, 24, 36, 48, and 72 Hz respectively. It can be seen from the coda that Q is changing with time. The early coda amplitude falls off rapidly while the late coda shows little decay.



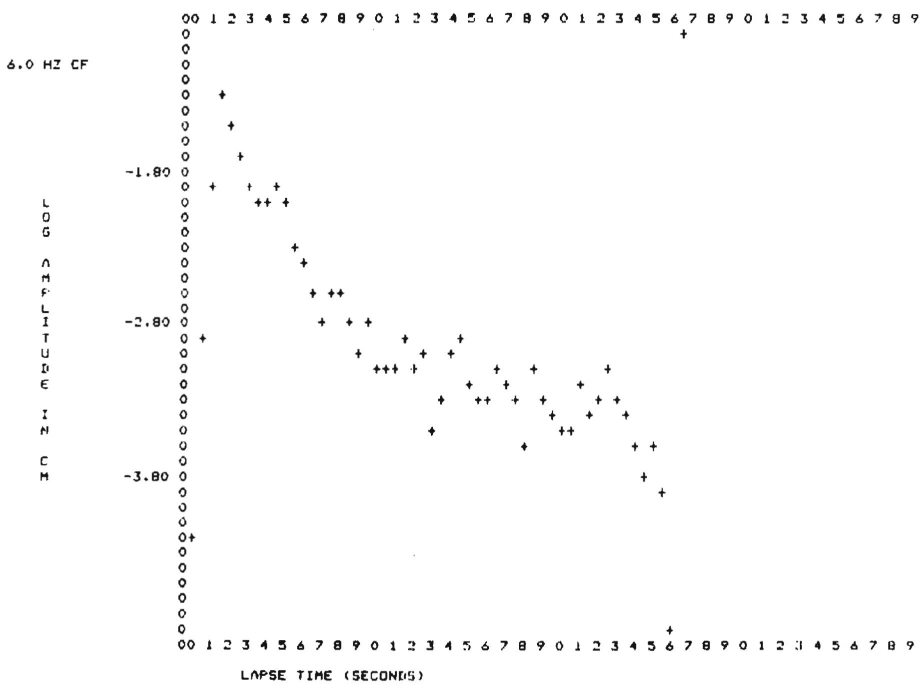
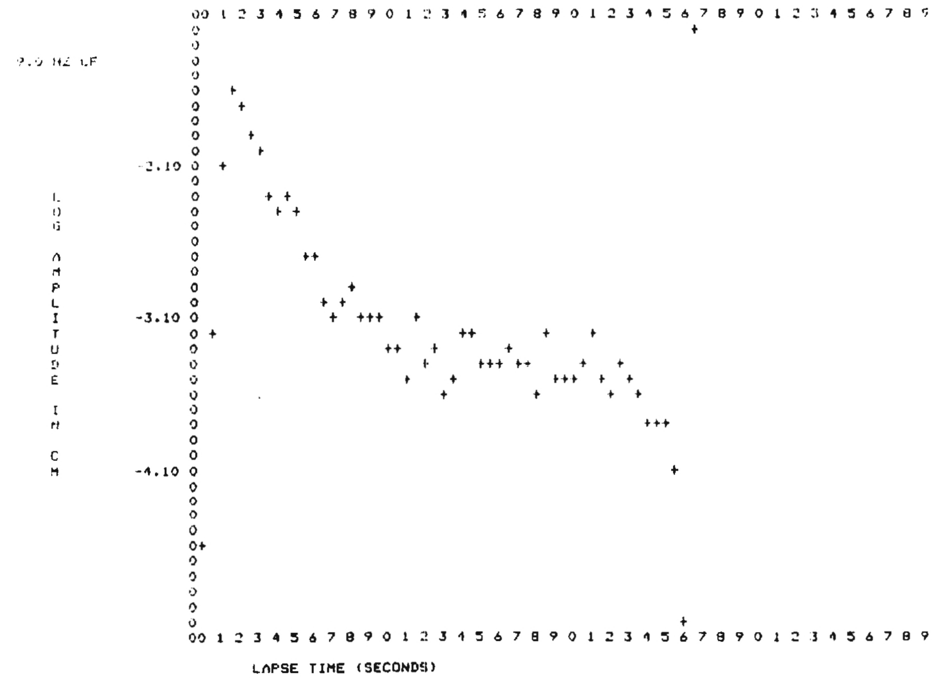


Figure 3. b.(top) c.(bottom)



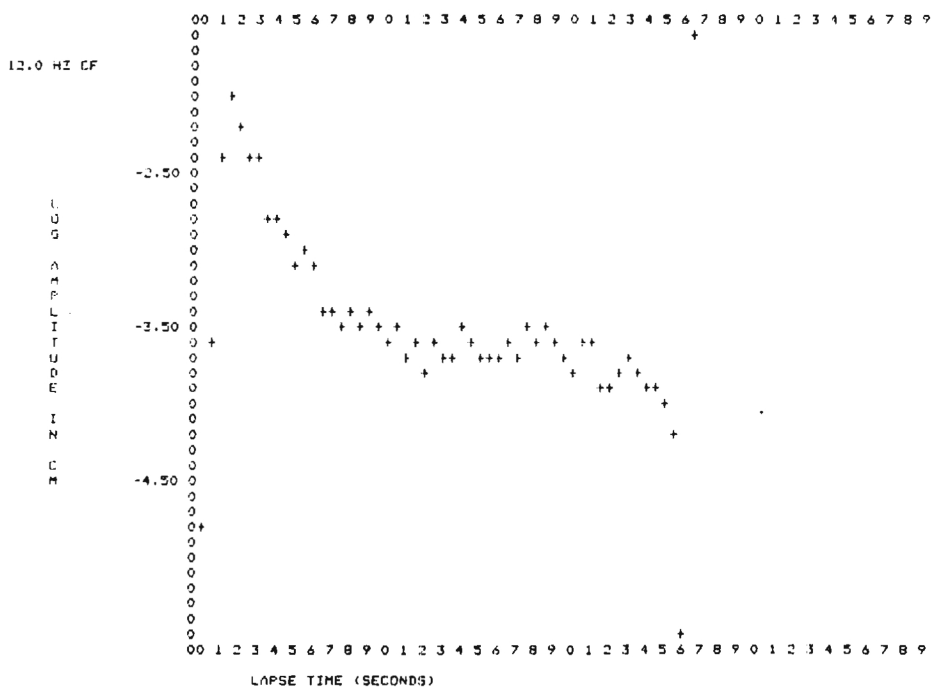
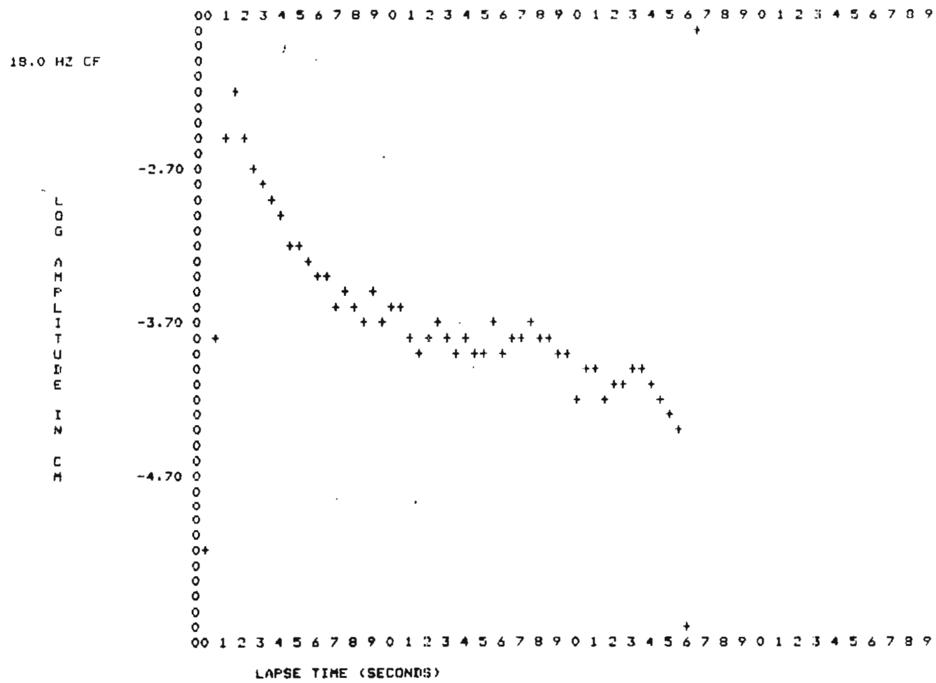


Figure 3. d.(top) e.(bottom)



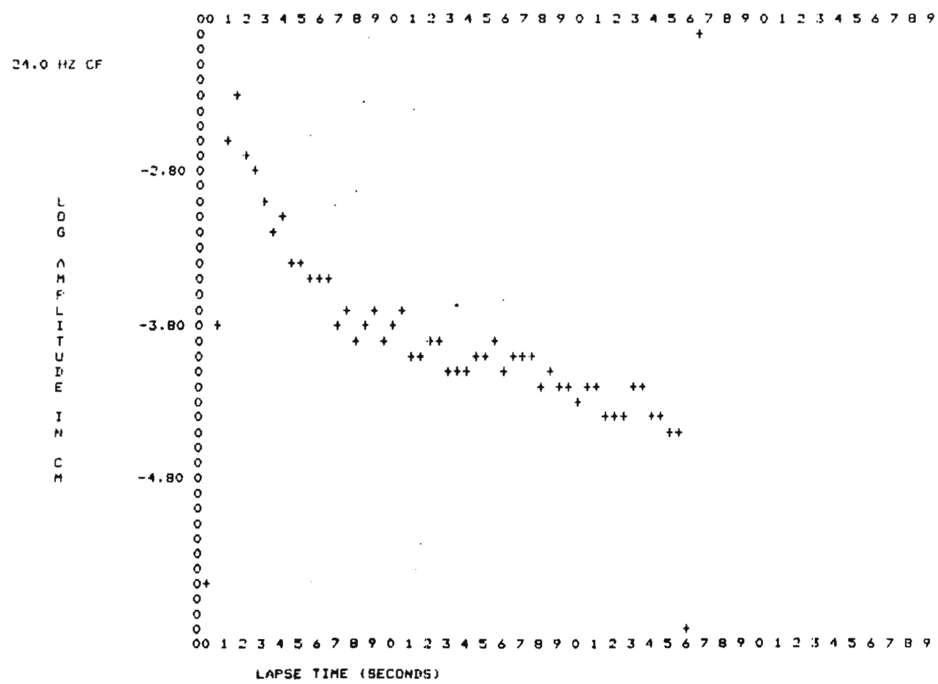


Figure 3 . f.(top) g.(bottom)

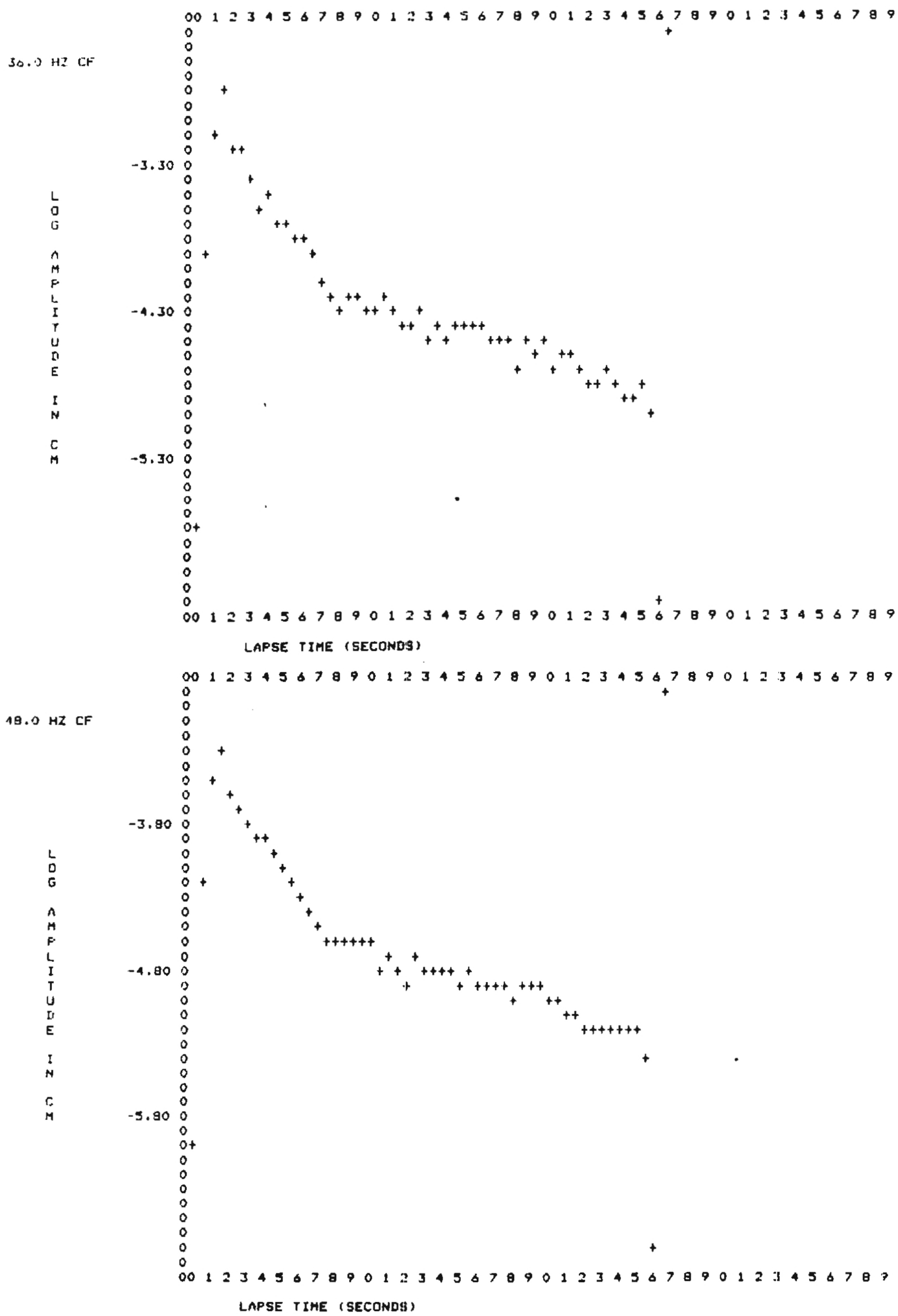


Figure 3. h.(top) i.(bottom)

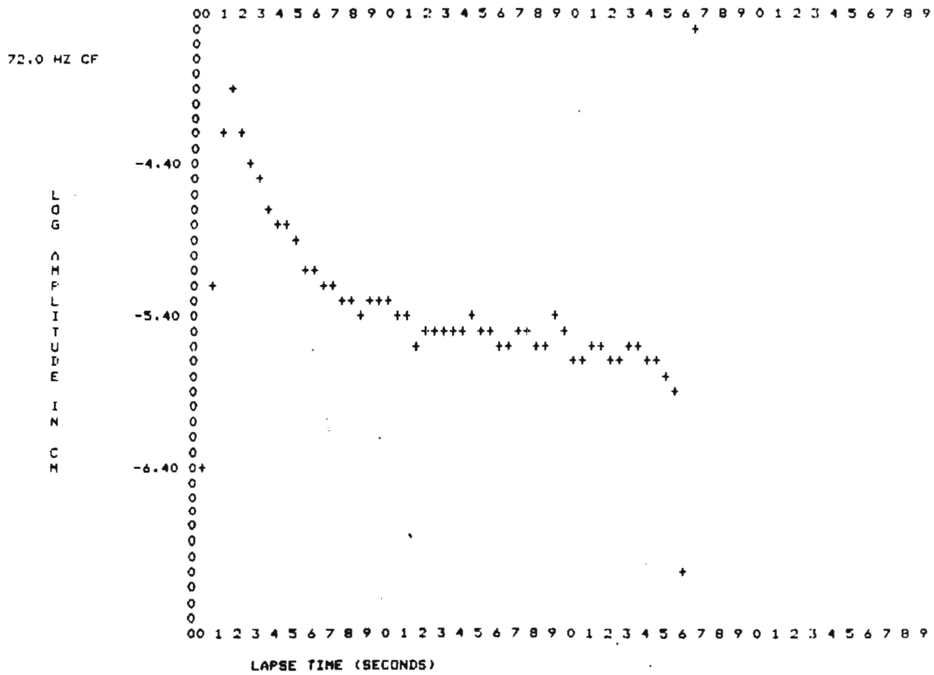


Figure 3. j.

The next step is to fit the model equation for $\log[A(f|t)]$ to the backscatter portion of the $\log[A(f|t)]$ plots for an estimate of Q_T. The method of least squares is used and two fits are made corresponding to the two choices of m = 1.0 and m = 0.5. Standard deviations are also calculated for each fit. Attempts to fit the model taking m as an independent variable produced erratic results as has been reported in the literature (Aki and Chouet, 1975).

The backscatter portion of the filtered data is determined as follows. Energy arriving after $\sqrt{2}$ times the primary S-wave travel time is backscatter in the sense that the scattering angle is greater than 90 degrees. However, because of the 1.28 second window and 0.5 second steps used for averaging only arrivals 1.0 second later than this are entirely backscatter.

On some records the dynamic range of the recording instrument has been exceeded for the early coda. The saturated portion of the coda is not considered in the analysis. Also excluded are the last three points of the filter output as they are distorted by artificial extension of the end of the trace necessary in filtering.

Another consideration is noise. The background noise level is assumed constant and taken equal to the peak noise level recorded prior to the P-wave arrival. If no noise is recorded here it is assumed to be + or - 0.5 times one unit of digitization. The noise level is converted to ground displacement amplitude. A reference, five times (14db above) this noise level, is taken as the minimum acceptable signal level. Portions of the coda for which the average signal amplitude falls below this reference are not considered in the fits.

Finally, from the longest records it is evident that Q_T changes with time causing the Q_T value obtained to depend on the portion of coda fit. This can be seen on the sample record in Figure 3. This time dependence of Q_T was also observed by Rautian and Khalturin (1978) who suggest variations of Q with depth and temporal variations of the geometric spreading factor as possible causes. The variation in spreading factor could result from a change in the type of waves being recorded as a function of time (e.g., surface to body waves as time increases). Another possibility is that of a significant deterministic influence on the energy arriving at the receiver. Long and Wilson (1982) have noticed resonance peaks in the power spectra of the Monticello records. The frequencies at which these occur are in congruence with conceivably supportable resonances in the Monticello reservoir. This can be regarded as an example of multiple scattering, which tends to raise the value of Q because energy previously scattered out of the coda is scattered back in, reducing the rate of decay. Indeed, in the limit of strong scattering, there is no contribution to Q from scattering (Dainty and Toksöz, 1977). Because of the observed temporal dependence of Q_T, fits are made to different portions of the coda where possible.

Results

Plots of the average over all stations and events of $\log[Q_T]$ versus log[frequency] are constructed and appear as Figure 4. Because a temporal dependence of Q_T was observed, three plots are made for each area and m value. They are from the beginning of backscatter to: I, five seconds beyond; II between five and nine seconds beyond; III,

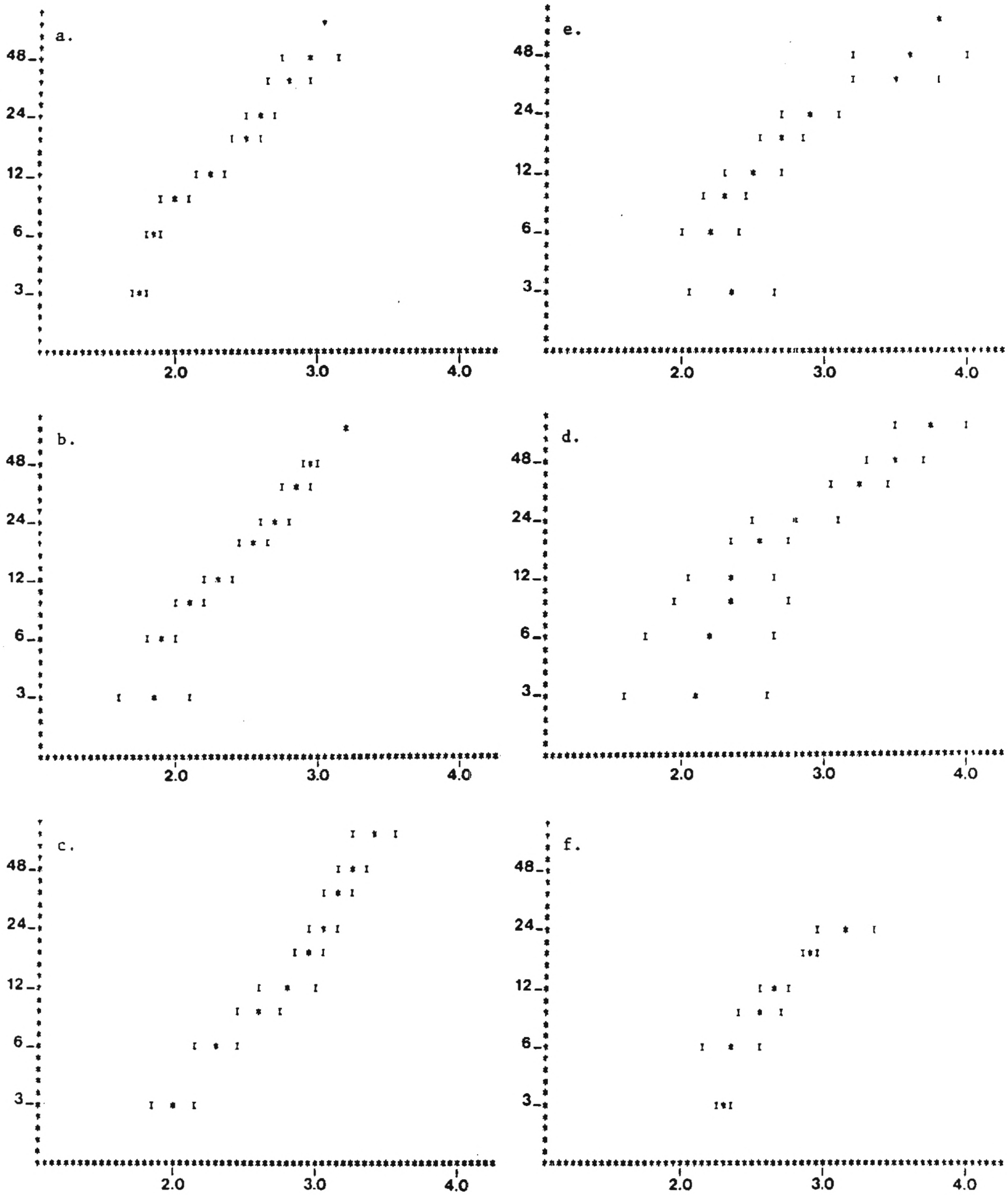


Figure 4a-f. Average log(Qtotal) versus Frequency, m=0.5. The vertical axes are frequency and the horizontal axes are log(Qtotal)
a, b, and c are for $t<5$, $5<t<9$, $9<t$ respectively at Monticello.
d, e, and f are for $t<5$, $5<t<9$, $9<t$ respectively at Mammoth.

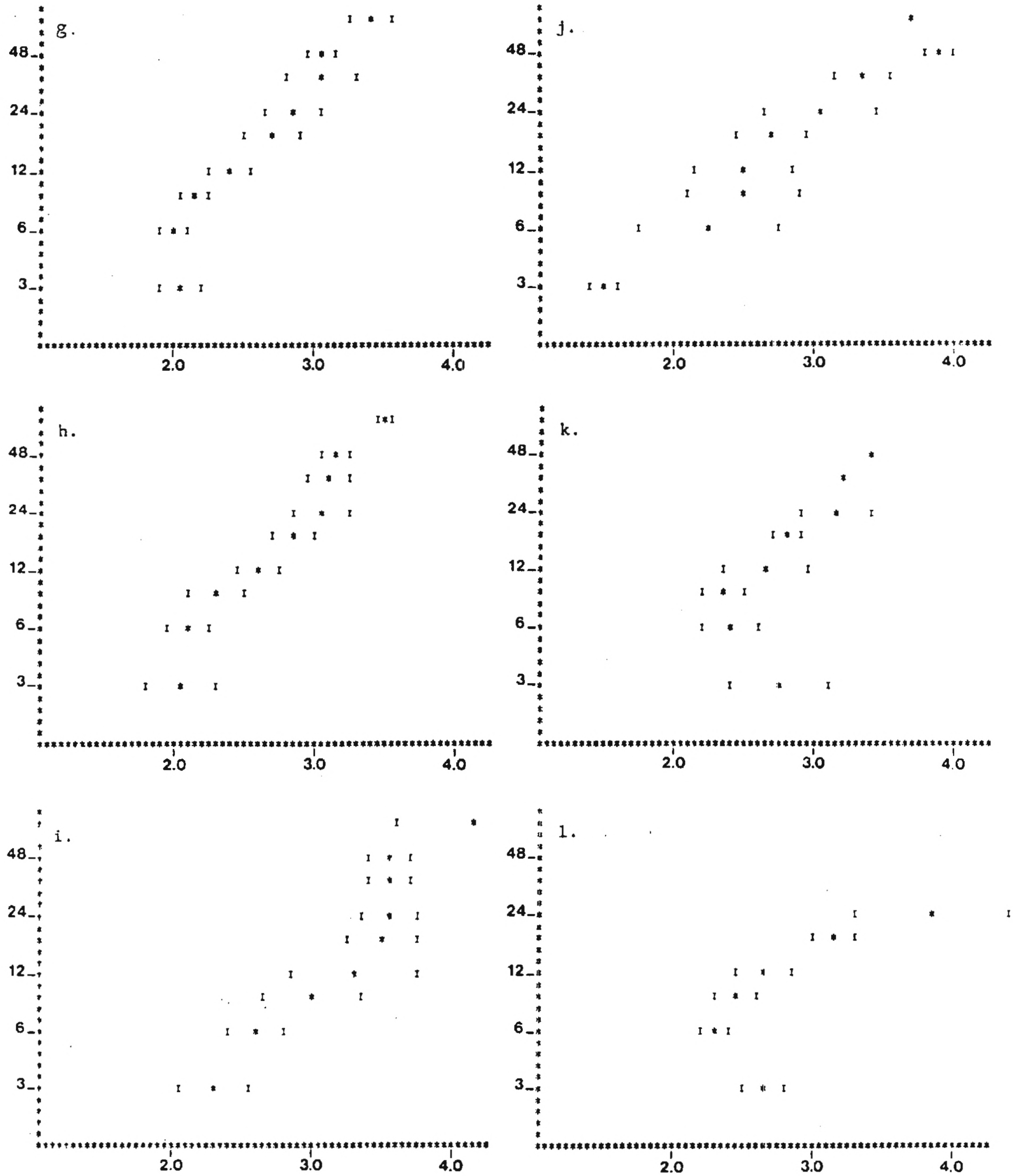


Figure 4g-1. Average log(Qtotal) versus Frequency, m=1.0. The vertical axes are frequency and the horizontal axes are log(Qtotal)
a, b, and c are for t< 5, 5<t< 9, 9<t respectively at Monticello.
d, e, and f are for t< 5, 5<t<9, 9<t respectively at Mammoth.

more than nine seconds beyond. These times were chosen arbitrarily within the limits of the data. As the plots are roughly linear, best fits of the equation $Q_T = af^b$ are made to each of these to determine the values of a and b. This approach is common in the literature. The a and b values obtained are presented in Table 3.

The frequency dependence of Q_T may be used to estimate Q_i and g if some assumptions are made about their frequency dependence. For the single backscatter model applied here the density of scatterers ρ is assumed constant. The backscatter cross section σ_b will be a constant for geometric (high frequency) scatter. The result is a constant g for geometric scatter with the single backscatter model. It is usually assumed that Q_i is independent of frequency in agreement with most laboratory measurements of Q_i (Knopoff, 1964). With these assumptions a least squares fit of $1/Q_T = 1/Q_i + gB/2\pi f$ is made to the Q_T versus frequency data to estimate g and Q_i (Dainty, 1981). Negative values of Q_i occur when the least square fit results in a $gB/2\pi f$ which is greater than $1/Q_T$. As Q_i increases with increasing $gB/2\pi f$ a negative Q_i result is indicative of a very high Q_i value. These results are shown in Table 4.

A second estimate of g is obtained as follows. From the least squares fit of log[A(f t)] estimates of Q_T and the source factor have been obtained. The filter output at the primary S-wave travel time at a given frequency is a component of the source factor C. This allows a second estimate of g from the primary S-wave spectra and the source factor $C = \log(s\Delta f)$.

Taking a result from the theory section

Table 3. Best Fit a and b Values. $Q_T = af^b$ at Monticello and Mammoth. m = 0.5 and m = 1.0 correspond to cylindrical and spherical spreading respectively.

	Monticello			Mammoth		
	a	b	freq	a	b	freq
<u>m = 0.5</u>						
I	6.3	1.2	6-48	1.5	1.8	3-24
I				6.2	0.6	12-72
II	9.7	1.1	3-72	4.1	1.6	6-72
III	31.8	1.0	3-72	16.6	1.3	6-24
<u>m = 1.0</u>						
I	9.2	1.2	6-72	9.0	1.4	3-36
II	11.0	1.2	6-72	20.0	1.2	6-48
III	29.6	1.5	3-24	7.6	1.7	6-24
III	2750	0	18-48			

Table 4. Log(g) Values From Fitting Equation (5).

	Monticello			Mammoth		
	log g	Q_i	freq	log g	Q_i	freq
<u>m = 0.5</u>						
I	-0.92	940	3-72	-1.28	1050	3-72
II	-1.00	900	3-72	-1.46	860	3-72
III	-1.19	*	3-72	-1.49	1050	3-24
<u>m = 1.0</u>						
I	-0.70	*	3-72	-1.17	1000	3-72
II	-1.92	610	3-72	-1.17	2960	3-48
III	-1.49	*	3-72	-1.82	580	3-24

* indicates Q_i approaching infinity.

$$s = \left|\phi\,(f\,|\,r_o)\right|^2 \, 4m\pi r_o^{2m} \, (B/2)^{1-2m} \, g \, . \tag{11}$$

Also, the average amplitude, $\left|\phi\,(f\,|\,r)\right|$, of waves incident at a scatterer at distance r is

$$\left|\phi(f\,|\,r)\right| = \left|\phi\,(f\,|\,r_o)\right| (r/r_o)^{-m} \exp(-\pi f r/BQ_T) \, . \tag{12}$$

It is important to note that $\left|\phi\,(f\,|\,r)\right|$ is also the average spectral amplitude of the direct S-wave where r is the source receiver distance. This is estimated from a linear interpolation of the spectral amplitudes measured in the two windows closest to the S arrival time. The interpolation is necessary as the window is generally not centered on the S arrival. Solving equation (12) for the source spectrum at r_o, $\left|\phi\,(f\,|\,r_o)\right|$, and substituting in (11) a solution for log(g) is obtained.

$$\log(g) = 2C - \log[4m\pi(B/2)^{1-2m} \left|\phi(f\,|\,r)\right|^2 r^{2m} \exp(2\pi f t/Q_T)\Delta f \,] \, . \tag{13}$$

Estimates of average log(g) are obtained using this equation for m = 0.5 and or m = 1 and are tabulated in Table 5.

Discussion

The $\log(Q_T)$ versus log(frequency) curves of Figure 4 show Q_T to be roughly the same for similar fits at Mammoth and Monticello. Though it is generally agreed that Q_T is higher in the eastern U.S. the results here may be explained by the short lapse times of the codas used to estimate Q_T. These lapse times are much shorter than are usually reported in the literature for Q_T from coda estimates.

Table 5. Log(g) as Estimated from the S-wave Spectrum and Coda Source Function.

	Monticello		Mammoth	
	log g	freq	log g	freq
m = 0.5				
I	-2.50	3-72	-1.90	3-72
II	-2.53	3-72	-1.78	3-72
III	-3.27	3-72	-2.36	3-24
m = 1.0				
I	-3.03	3-72	-2.46	3-72
II	-2.96	3-72	-2.26	3-48
III	-3.50	3-72	-2.77	3-24

This implies that the results here represent the near surface Q_T and that it is approximately the same for both regions.

The results presented in Table 3 are the a and b values obtained by best fit of $Q_T = af^b$ to the log[Q_T] versus log[frequency] curves of Figure 5. The values of b in Table 3 range from 0.6 to 1.8, with an average of 1.3. The usual value of b reported in the literature is around 0.5 (0.6, Fedotov and Boldyrev, 1969; 0.5, Rautian and Khalturin, 1978; 0.5, Tsujiura, 1978; 0.6, Aki, 1980). However, the higher values obtained here are not unique. Rovelli (1982) has reported a b value of 1.1 for Friuli, Italy. This value was determined by coda analysis. Aki suggests that the b value, 0.8, which he determined for northeast Kanto, Japan, is the result of increased scattering by a higher density of scatterers associated with the higher level of tectonic activity in this part of the region. Aki and Chouet (1975) report on Q versus frequency at Stone Canyon, California. A b value of 0.9 is obtained by best fit of $Q_T = af^b$ to the reported results. Stone Canyon, Mammoth Lakes, and Friuli are all highly active regions.

The b value results at Monticello may be considered anomalous for this region of relative tectonic stability. However, in addition to including the reservoirs as a substantial part of the region sampled the codas and hypocentral distances at Monticello are much shorter than are usually reported in the literature for other studies. Consequently the near surface scatterers are favored in sampling. Andrews (1982) has suggested that near surface scattering is the dominant scattering effect for the similar situation at Mammoth Lakes.

Table 4 gives the results of fitting equation (5) to the values of Q_T as a function of frequency. Q_i and g are considered constants with frequency in making this fit. In (5), the two terms on the right hand side will be equal for a frequency F given by

$$F = gBQ_i/(2\pi) \tag{14}$$

Table 6 gives the values of F found using the results presented in Table 4. For frequencies less than F, scattering dominates the attenuation, while for frequencies greater than F, inelastic attenuation will dominate. From Table 6, scattering dominates attenuation over most of the frequency range in most cases, especially at Monticello. From (5), if scattering dominates then Q_T should be proportional to frequency, in approximate agreement with the results presented in Table 3. The proportionality of Q_T with frequency is also seen in Figure 4.

There is, however, another issue raised by Table 6. Dainty and Toksoz (1981) propose that if the frequency $f < F$, multiple scattering is a possibility. Obviously this situation holds for most of the codas considered here, from Table 6. Dainty and Toksoz point out that this seems to be true for many cases cited in the literature. However, Dainty and Toksoz's condition is only a necessary, not a sufficient, condition for multiple scattering. Since it will take some time after the origin time for multiple scattering to become important, a condition from Sato (1977) that multiple scattering will become important for times $t > T$, where $T = 1/(gB)$, is appropriate; if $f < F$ and $t > T$, strong scattering is probably occurring. T is also

Table 6. $F = gBQ_i/(2\pi)$, $T = 1/(gB)$. * is F tending to infinity.

	Monticello			Mammoth		
	F (Hz)	T (sec)	Freq. Range (Hz)	F (Hz)	T (sec)	Freq. Range (Hz)
<u>m = 0.5</u>						
I	60	2.4	3-72	30	5.4	3-72
II	50	2.9	3-72	15	8	3-72
III	*	4.4	3-72	20	9	3-24
<u>m = 1.0</u>						
I	*	1.4	3-72	40	4.2	3-72
II	4	24	3-72	100	4.2	3-48
III	*	9	3-72	5	19	3-24

given in Table 6, and in general indicates that multiple scattering might be occurring in some of the codas, remembering that I indicates $t \leq 5$ sec; II, $t \leq 9$ sec; III, $t > 9$ sec; but in practice t is less 30 sec for all codas in III. More generally, both F and T in Table 6 are of the same order of magnitude as the frequencies and times considered in this study, suggesting that the situation may be transitional between weak and strong scattering. A similar conclusion was reached by Dainty and Toksöz (1981). Theoretical formulations to handle this problem (e.g., Gao et al., 1983) do not appear to conserve energy and do not appear to agree with the strong scattering diffusion formalism of Dainty and Toksöz (1977).

The log[g] values obtained from the coda source spectrum are presented in Table 5. In general they are about an order of magnitude to two orders of magnitude smaller than the log[g] values obtained from Dainty's equation. Andrews (1982) has found the source spectral amplitude from the coda to be ten times larger than the spectral amplitude measured from the S-wave. As the value of g here is proportional to the quotient (coda source spectrum/S wave source spectrum), a small g value means our results do not agree with those of Andrews'. However, they are in agreement with the theoretical results reported in the second part of this report.

Throughout this report, the question of whether the scattering is two-dimensional ($m = 0.5$, cylindrical spreading, surface waves) or three-dimensional ($m = 1.0$, spherical spreading, body waves) has been left open. As discussed in the section on Analysis, it was not possible to determine the correct value of m from the data. Aki

(1980) has suggested on various grounds that the three-dimensional case is probably appropriate, but he dealt with much longer codas. Most of the more important results presented here seem to hold for both cases. It must be conceded, however, that the change in coda decay as time increases could be due to a change in Q_T reflecting the change from surface waves to body waves.

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HIGH FREQUENCY ACOUSTIC BACKSCATTERING AND SEISMIC ATTENUATION

By Anton M. Dainty

Introduction

The phenomenon of scattering of seismic waves has attracted increasing attention in recent years. Backscattering of seismic waves has been used to explain terrestrial codas (Aki and Chouet, 1975; Dainty and Toksoz, 1981) and part or all of the attenuation suffered by seismic waves at frequencies of 1 Hz and greater (Aki, 1980; Dainty, 1981; Kikuchi, 1981; Wu, 1982). To analyse observed data on attenuation, coda amplitudes and other manifestations of scattering, a model of the earth as a scattering medium is required together with a theory to derive quantitative results from this model. Two general types of model have been used. One is the "randomly distributed specific scatterers" model. This model assumes scatterers of a specific type, for example spheres, randomly distributed throughout an otherwise homogeneous region. Examples of this type of model are the sphere model of Dainty (1981) used to explain the frequency dependence of the attenuation parameter Q at frequencies above 1 Hz, and the crack model of Kikuchi (1981), also used to analyse Q. A second type of model is the "random medium" model, in which the earth is considered to have average properties that do not vary from place to place, but with random fluctuations of the material properties. Examples are the use of an acoustic random medium by Aki (1980) and Wu (1982) to analyse Q above 1 Hz.

In this paper I shall examine the acoustic random medium model using the results of Chernov (1960) in the limit of high frequency backscattering. This problem was chosen because investigators using this model have generally concluded that this is the practical case in

the earth for frequencies of 1 Hz and higher. By analysing the high frequency limit, insight may be gained as to what type of information about the medium may be deduced from high frequency data. An additional benefit is an understanding of the relationship between the two types of models that have been used to examine this problem.

Random Medium Models

The simplest type of random medium used in seismology is the acoustic random medium discussed by Chernov (1960) (a summary of some useful results is given in Aki and Richards, 1980). The medium is considered to have fluctuations of velocity and density about mean values; for simplicity I will consider only the velocity fluctuations. Let the acoustic velocity C be given by

$$C = C_0(1-\mu(\vec{x})) \tag{1}$$

where C_0 is a constant, and $\mu(\vec{x})$, the slowness perturbation, is a random function of position $\vec{x}$ and has an average value of zero. The autocorrelation $N(\vec{r})$ of the slowness perturbation μ is then

$$N(\vec{r}) = \langle\mu(\vec{x})\mu(\vec{x}+\vec{r})\rangle/\langle\mu^2\rangle \tag{2}$$

The vector $\vec{r}$ is the lag, and the symbol $\langle\rangle$ means averaging over all $\vec{x}$. If the fluctuation is isotropic, then

$$N(\vec{r}) = N(r) \tag{3}$$

i.e., the autocorrelation depends only on the magnitude of the lag and not its direction.

Chernov (1960) finds the amplitude of singly (weakly) scattered waves for the case of a plane wave travelling through an acoustic medium of velocity C_0 without velocity fluctuations, except for a volume V containing velocity fluctuations as described above. If the incident (pressure) wave is given by

$$p_I = A \cdot \exp[2\pi fi(t-x/C_0)] \tag{4}$$

where f is the frequency, then the amplitude of the scattered wave at position $\vec{R}$ from the center of V in the limit of $\vec{R}$ large compared with the linear dimension of V is given by

$$|p_s|^2 = \frac{4\pi^2 VA^2 f^3 \langle\mu^2\rangle}{C_0^3 R^2 \sin(\theta/2)} \cdot \int_0^\infty N(r) \cdot \sin[(4\pi fr \cdot \sin(\theta/2))/C_0] \cdot rdr \tag{5}$$

(equation 45, p. 51, Chernov, 1960). The scattering angle θ is the angle between $\vec{R}$ and the direction of propagation; I shall define "back-scattering" as the case $\theta > \pi/2$. Equation (5) was first derived, in a slightly different form, by Pekeris (1947). This paper is concerned with the evaluation of (5) in the limit of large f and $\theta > \pi/2$.

Before proceeding, it should be noted that the acoustic medium described above is obviously too simple to describe the earth, which is

an elastic medium presumably containing fluctuations of both elastic constants and density, and through which two types of waves, compressional and shear, can propagate. Knopoff and Hudson (1967) have demonstrated that converted waves (scattered compressional waves from incident shear waves, and vice versa) are not important at high frequencies. Haddon and Cleary (1974) examined the problem of scattering from an inhomogeneous elastic medium; some of their (unpublished) results are discussed in Aki and Richards (1980). These results indicate that at least some of the equivalent formulas to equation (5) are of a similar form. Accordingly, equation (5) may be considered typical of integrals that appear in this type of problem.

<u>Evaluation of the Backscattered Intensity at High Frequency</u>

To evaluate (5), we need to find the integral

$$\int_0^\infty N(r) \cdot \sin[(4\pi fr \cdot \sin(\theta/2))/C_0] \cdot rdr = F(\infty) - F(0) \qquad (6)$$

where F is the indefinite integral. However, if there is a scale length associated with the fluctuations, N(r) will generally decline rapidly for r larger than a. Accordingly, I will assume that $F(\infty) \to 0$ in (6); this will certainly be true of $N(r) \to \exp[-r/a]$ for large r. Then

$$\int_0^\infty N(r) \cdot \sin[(4\pi fr \cdot \sin(\theta/2))/C_0] \cdot rdr \cong -F(0) \qquad (7)$$

To approximate F(0) in the case of f large and $\theta > \pi/2$, note that for this case the sine term in (7) varies much more rapidly than N(r), assuming N is a smooth function. Accordingly, I will expand N(r) in a Taylor series near r = 0. First, however, define

$$u = [4\pi fr \cdot \sin(\theta/2)]/C_0 \qquad (8)$$

and

$$K = [4\pi f \cdot \sin(\theta/2)]/C_0 \qquad (9)$$

Then (7) becomes

$$\frac{1}{K^2} \int_0^\infty N(\tfrac{u}{K}) \cdot \sin(u) \cdot udu \cong -F(0) \qquad (10)$$

Expand $N(\frac{u}{K})$ as, remembering that $u/K = r$,

$$N(\frac{u}{K}) = N(0) + \frac{u}{K}\left(\frac{dN}{dr}\right)_{r=0} + \frac{1}{2}\left(\frac{u}{K}\right)^2\left(\frac{d^2N}{dr^2}\right)_{r=0} + \cdots \tag{11}$$

Then

$$F(u) \cong \frac{1}{K^2}\left[N(0)\int u \,.\, \sin(u) \,.\, du + \frac{1}{K}\left(\frac{dN}{dr}\right)_{r=0} \int u^2 \sin(u) \,.\, du \right.$$

$$\left. + \frac{1}{2K^2}\left(\frac{d^2N}{dr^2}\right)_{r=0} \int u^3 \sin(u) \,.\, du + \ldots.\right] \tag{12}$$

To evaluate (12), use the tabulated integrals (Carmichael and Smith, 1962):

$$\int u \,.\, \sin(u) \,.\, du = \sin(u) - u \,.\, \cos(u) \tag{13}$$

$$\int u^m \sin(u) \,.\, du = -u^m \cos(u) + m\int u^{m-1}\cos(u) \,.\, du \tag{14}$$

$$\int u^m \cos(u) \,.\, du = u^m \sin(u) - m\int u^{m-1}\sin(u) \,.\, du \tag{15}$$

$$\int u \,.\, \cos(u) \,.\, du = \cos(u) + u \,.\, \sin(u) \tag{16}$$

Using (14) and (16),

$$\int u^2 \sin(u) \,.\, du = 2u \,.\, \sin(u) + (2 - u^2)\cos(u) \tag{17}$$

Using (13), (14), (15), and (16)

$$\int u^m \sin(u) \,.\, du = \cos(u) \,.\, \left(-u^m + \sum_{n=1}^{\frac{m-1}{2}} (-1)^{n+1} m(m-1)\ldots(m-2n+1)u^{m-2n}\right) \qquad (m \text{ odd}) \quad (18)$$

$$+ \sin(u)\left(mu^{m-1} + \sum_{n=1}^{\frac{m-1}{2}} (-1)^n m(m-1)\ldots(m-2n)u^{m-2n-1}\right)$$

$$= \cos(u)\left(-u^m + \sum_{n=1}^{m/2} (-1)^{n+1} m(m-1)\ldots(m-2n+1)u^{m-2n}\right) \qquad (m \text{ even}) \quad (19)$$

$$+ \sin(u)\left(mu^{m-1} + \sum_{n=1}^{m/2} (-1)^n m(m-1)\ldots(m-2n)u^{m-2n-1}\right)$$

From (13), (17), (18), and (19), as $u \to 0$,

$$\int u \,.\, \sin(u) \,.\, du \to 0 \qquad (20)$$

$$\int u^2 \sin(u) \,.\, du \to 2 \qquad (21)$$

$$\int u^m \sin(u) \,.\, du \to 0 \qquad (m \text{ odd}) \qquad (22)$$

$$\to (-1)^{m/2+1} m! \quad (m \text{ even}) \qquad (23)$$

Substituting in (12) and letting $u \to 0$,

$$F(0) = \sum_{n=1}^{\infty} \frac{1}{K^{2n+1}} (-1)^{n+1} \frac{(2n)!}{(2n-1)!} \left(\frac{d^{2n-1}N}{dr^{2n-1}}\right)_{r=0} \qquad (24)$$

According to (24), under the conditions described the backscattered intensity depends only on the odd derivatives of N at r=0, and not on the even derivatives. Since this is a high frequency approximation, the first term in (24) will be the most important, i.e.,

$$F(0) \cong \frac{2}{K^3} \left(\frac{dN}{dr}\right)_{r=0} + O(1/K^5) \tag{25}$$

Substituting in (10) and (5)

$$|p_s|^2 = - \frac{VA^2\langle\mu^2\rangle}{8\pi R^2 \sin^4(\theta/2)} \left(\frac{dN}{dr}\right)_{r=0} + O(1/f^2) \tag{26}$$

This indicates that the backscattered intensity is independent of frequency, provided that the autocorrelation has a non-zero derivative at zero lag. Since the first derivative at zero lag must be negative ($N(r)$ has a maximum at $r=0$), $|p_s|^2$ will be positive.

To test equation (26) I will compare it to two specified cases of $N(r)$ for which explicit formulas for $|p_s|^2$ are available (Chernov, 1960). If

$$N(r) = \exp(-r/a) \tag{27}$$

then

$$|p_s|^2 = \frac{32\pi^3 VA^2 f^4 a^3 \langle\mu^2\rangle}{R^2(C_0{}^4 + 16\pi^2 f^2 a^2 \sin^2(\theta/2))^2} \tag{28}$$

$$\rightarrow \frac{VA^2\langle\mu^2\rangle}{8\pi R^2 a \,.\, \sin^4(\theta/2)} \qquad (f \rightarrow \infty,\ \theta \geq \pi/2) \tag{29}$$

The approximate solution given by (26) reproduces this result.

Another case for which an exact solution is available is

$$N(r) = \exp(-r^2/a^2) \tag{30}$$

Then

$$|p_s|^2 = \frac{4\pi^{7/2} V A^2 f^4 a^3 \langle\mu^2\rangle}{R^2} \exp[-(4\pi^2 f^2 a^2 \sin^2(\theta/2))/C_o] \tag{31}$$

$$\rightarrow 0 \qquad (f \rightarrow \infty,\ \theta \geq \pi/2) \tag{32}$$

Since $N(r)$ has no non-zero derivatives of odd order for $r=0$, this is also in accord with (26).

Discussion

The limit of validity of (26) can be obtained from (7), since we require that the sine terms in (7) vary much more rapidly than N(r). If a is a scale length describing the range of r over which N(r) varies appreciably, we must have

$$(4\pi fa \cdot \sin(\theta/2))/C_0 >> 2\pi \tag{33}$$

or

$$(2fa \cdot \sin(\theta/2))/C_0 >> 1 \tag{34}$$

Note that (34) requires that the scattering angle θ cannot be small; in fact,

$$\sin(\theta/2) >> C_0/(2fa) \tag{35}$$

is necessary. Thus the approximation used here is not valid for forward scattering. It may, however, be valid for $\theta < \pi/2$ provided the restriction (35) is observed. Within these limits, the scattered intensity varies as $1/(\sin^4\theta/2)$, indicating that "side scattering" ($\theta \sim \pi/2$) is about four times as strong as "back scattering" ($\theta \sim \pi$) in the strict sense.

An important application of the theory of backscattering is the interpretation of the frequency variation of Q for shear waves in the range 1-30 Hz (Aki, 1980; Wu, 1982). Q, the quality factor, describes the attenuation of the amplitude A in (4) by

$$A(x) = A(0)\exp[-\pi fx/(QC_0)] \tag{36}$$

The total Q represents the effects of energy loss from the primary wave by the combined mechanisms of loss of seismic energy to heat (Q_i) and loss due to backscattering (Q_s) according to the relation (Warren, 1972):

$$1/Q = 1/Q_i + 1/Q_s \tag{37}$$

Wu (1982) derives an expression for Q_s, with the proper choice of V, as

$$1/Q_s = \frac{C_0R^2}{2\pi fA^2V} \int_0^{2\pi} d\phi \int_{\pi/2}^{\pi} d\theta \left|p_s\right|^2 \sin(\theta) \tag{38}$$

where ϕ is an azimuthal angle about the direction of propagation of the primary wave and the integral over scattering angle θ is taken over all backscattering angles.

Substituting for $\left|p_s\right|^2$ from (26) and performing the integral over ϕ, in the limit of high frequency,

$$1/Q_s = -\frac{C_0\langle\mu^2\rangle}{8\pi f} \left(\frac{dN}{dr}\right)_{r=0} \int_{\pi/2}^{\pi} \frac{\sin(\theta)d\theta}{\sin^4(\theta/2)} \tag{39}$$

This may be evaluated using the trigonometric identity

$$2\sin^2(\theta/2) = 1 - \cos(\theta) \tag{40}$$

as

$$1/Q_s = -\frac{C_o\langle\mu^2\rangle}{4\pi f}\left(\frac{dN}{dr}\right)_{r=0} \tag{41}$$

Comparing this with an expression from Dainty (1981),

$$1/Q_s = gC_o/(2\pi f) \tag{42}$$

then

$$g = -\frac{\langle\mu^2\rangle}{2}\left(\frac{dN}{dr}\right)_{r=o} \tag{43}$$

This indicates that g, the turbidity, is independent of frequency and depends on the product of the mean square slowness fluctuation and the derivative of the autocorrelation of the slowness fluctuation. The lack of dependence of g on frequency is the same result obtained by Dainty (1981) for a set of randomly distributed spheres in geometric scatter; thus we may say that the approximation explored in this paper is the analog of geometric scattering from obstacles. Indeed, observationally we cannot distinguish between these cases, nor can we distinguish between different autocorrelation functions N(r) except insofar as their derivatives at r=0 differ.

In (38), the integral over the scattering angle was taken over all backscattered angles. Sato (1982a) has used a different approach (the mean wave method) from that used by Chernov (1960) and Wu (1982). His work suggests that instead of (38),

$$1/Q_s = \frac{C_o R^2}{2\pi f A^2 V} \int_0^{2\pi} d\phi \left[\int_{\theta_c}^{\pi} d\theta \left|p_s\right|^2 \sin(\theta) + \int_0^{\theta_c} d\theta \left|p_s\right|^2 \sin^4(\theta/2)\sin(\theta) \right] \tag{44}$$

$$\sin(\theta_c/2) = 1/4, \quad \theta_c \cong 30° \tag{45}$$

Comparing (44) and (38), Sato (1982a) indicates that all energy scattered at angles greater than θ_c should be considered lost from the primary wave (the first integral in the brackets) and a fraction of the more forward scattered energy should also be excluded (the second integral). Wu (1982) excluded only the backscattered energy. To evaluate (44) I shall use (26); while (26) is not valid near $\theta=0$, the presence of the weighting factor $[\sin^4(\theta/2)\sin(\theta)]$ in (44) ensures that the integral is small in this region. Then

$$1/Q_s \cong -\frac{7}{2\pi}\frac{C_o\langle\mu^2\rangle}{f} \left(\frac{dN}{dr}\right)_{r=0} \tag{46}$$

from (42), this gives

$$g \cong -7\langle\mu^2\rangle \left(\frac{dN}{dr}\right)_{r=0} \tag{47}$$

The value in (47) is larger than that in (43) because a greater part of the scattered energy was considered lost from the primary wave.

The results in (43) and/or (47) may be compared with another parameter, the backscattering turbidity

$$g_\pi = \frac{R^2}{AV} \left| p_s(\pi) \right|^2 \tag{48}$$

$$= \frac{\langle \mu^2 \rangle}{8\pi} \left(\frac{dN}{dr}\right)_{r=0} \tag{49}$$

This parameter controls the amplitude level of coda waves relative to the direct wave, after correction for geometrical spreading and attenuation (Aki and Chouet, 1975). Aki (1980) suggested that $g_\pi \sim g$; Sato (1982b) presented a more detailed calculation. According to the formulas presented here,

$$g_\pi/g = 1/(4\pi) \cong 10\% \tag{50}$$

if expression (43) is used for g,

$$g_\pi/g \cong 1/(56\pi) \cong 1\% \tag{51}$$

if (47) is used for g.

A final, important question has to do with the nature of the slowness fluctuation $\mu(\vec{x})$ that will lead to an autocorrelation N(r) that has a non-zero first derivative at r=0. Chernov (1960) asserts that the first derivative of N(r) must tend to zero for $r \to 0$ if $\mu(\vec{x})$ is continuous, although a proof is only given for the one-dimensional case. This is a special case of a result due to Taylor (1920), who proved (for the one-dimensional case) that the autocorrelation of a continuous function has only even-ordered derivatives that are non-zero at zero

lag. Pekeris (1942) comments further on Taylor's result. However, K. Aki (personal communication) has pointed out that the (one-dimensional) function

$$\mu(x) = \int_{-\infty}^{\infty} w(\tau)h(x-\tau)d\tau \tag{52}$$

$$h(x) = \exp[-x/a] \qquad x \geq 0 \tag{53}$$

$$= 0 \qquad x < 0$$

has the autocorrelation given by (27) if w(x) is any function with a power spectrum that is constant with frequency. If w(x) is chosen to be "white noise", it appears that (52) is continuous, even though it has an autocorrelation which has a non-zero first derivative at zero lag. However, the first derivative of (52) is

$$\mu'(x) = \int_{-\infty}^{\infty} w(\tau)h'(x-\tau)d\tau$$

$$= \int_{-\infty}^{\infty} w(\tau)[\delta(x-\tau) - (1/a)h(x-\tau)]d\tau$$

$$= w(x) - (1/a)\mu(x) \tag{54}$$

This is a discontinuous function, since w(x) is discontinuous. Thus the function $\mu(x)$ is "continuous" but is not "smooth". This property of $\mu(x)$ is perhaps the reason that Taylor's (1920) and Chernov's (1960) proofs fail. The lack of smoothness may explain why the results

obtained here are equivalent to geometrical scattering from obstacles, since in this case the scattering is controlled by the discontinuity which is the boundary of the obstacle.

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"Influence of Scattering on Q in the Lithosphere"

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Two shallow earthquakes in the Mammoth Lakes, California, region have been analyzed to determine scattering parameters and their influence on attenuation. Two parameters were determined as a function of frequency from the coda of these events over the frequency range 3 to 72 Hz: the quality factor Q from coda decay and the backscattering turbidity, or backscattering cross-section per unit volume, from the ratio of the power spectrum of the coda to the square of the Fourier transform of the direct S wave. A single scattering theory of the coda due to Aki was used to estimate both the backscattering turbidity and the Q, although the results for Q are not likely to be highly model dependent. The results were interpreted in terms of high frequency scattering ("geometrical scattering") from either spheres or a "rough" random medium.

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19. ABSTRACT (Continued)

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Various possibilities have been examined to explain the discrepancies. There are three major categories: the measurements are not correct or are not being interpreted correctly; different medium properties or scatterers are responsible for backscattering and attenuation; or multiple scattering, not single scattering, is occurring in the coda. The last possibility seems the most likely, and indicates that parameters derived from coda analysis using single scatter theory should be treated with caution. We hope to extend this analysis to other areas.

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INVESTIGATION OF BACKSCATTERING AND CODA DECAY FOR SHALLOW EARTHQUAKES AT MAMMOTH LAKES, CALIFORNIA

By

Anton M. Dainty and An Tie

Introduction

In this report, we examine the codas of aftershocks of the May 25 and May 27, 1980, earthquakes at Mammoth Lakes, California. These aftershocks were digitally recorded by the United States Geological Survey (Archuleta et al., 1982). Our purpose in analyzing the codas of these earthquakes is to determine the attenuation quality factor, Q, from the coda decay, and the backscattering turbidity, $g(\pi)$, from the ratio of the coda spectral power to the direct S wave spectral power. By determining these two parameters we hope to independently find the contributions of scattering and anelasticity to Q. Some aftershocks from this sequence have already been examined in a previous report (Dainty and Duckworth, 1983; Appendix I) to determine Q as a function of frequency using coda decay, as well as a set of reservoir induced earthquakes at Monticello Reservoir, South Carolina.

The general theory of the coda power spectrum as a function of time has been covered in Dainty and Duckworth (1983), which has been included in this report as an Appendix. Briefly, the coda, which is that portion of the seismic energy which arrives after the normal body and surface wave arrivals have passed, is modeled for local earthquakes as consisting of singly backscattered S waves (Aki, 1980). The power spectrum between time (after event origin) t and $t+\Delta t$ is

$$P_c(\omega/t) = P_s(\omega) \cdot [\exp(\omega t_s/Q)] \cdot 8\pi \cdot V \cdot g(\pi,\omega) \cdot (t_s/t)^2 \cdot \exp(-\omega t/Q) \tag{1}$$

where P_s is the square of magnitude of the S wave Fourier transform as observed at the station, t_s is the direct S wave travel time, Q is the quality factor for S waves, V is the S wave velocity, and $g(\pi,\omega)$ is the backscattering turbidity, or backscattering cross-section per unit volume, appropriate for the crust. The duration of the S wave pulse is assumed to be shorter than Δt. Equation (1) may be rewritten as

$$P_c(\omega,t) = A \cdot t^{-2} \cdot \exp(-\omega t/Q) \tag{2}$$

where

$$A(\omega) = P_s(\omega) \cdot [\exp(\omega t_s/Q)] \cdot 8\pi \cdot V \cdot g(\pi,\omega) \cdot t_s^2 \tag{3}$$

A fit of $P_c(\omega,t)$ using an equation of the form of (2) will yield values of the quality factor Q for the coda decay and A, which is a measure of the size of the coda. From (3), we may obtain the backscattering turbidity $g(\pi,\omega)$ as

$$g(\pi,\omega) = A(\omega) \cdot [\exp(-\omega t_s/Q)] / [P_s(\omega) \cdot 8\pi \cdot V \cdot t_s^2] \tag{4}$$

Thus the backscattering turbidity at any frequency is proportional to the "size" of the coda ($A(\omega)$) divided by the "size" of the S wave ($P_s(\omega)$). The backscattering turbidity describes the amount of energy backscattered, in the sense of being returned along the direction of propagation of a wave, per unit volume per unit solid angle.

To analyze the data further, a model of the quality factor, Q, is required. Following Dainty (1981), we write

$$1/Q = 1/Q_i + 1/Q_s \tag{5}$$

$$1/Q_s = g_t(\omega) \cdot V/\omega \tag{6}$$

In (5), Q has been separated into Q due to anelasticity (Q_i, intrinsic atenuation) and Q due to scattering (Q_s). The expression for Q_s in (6) assumes single scattering and gives Q_s in terms of $g_t(\omega)$, the total turbidity, or total cross-section/unit volume. The total turbidity describes the total amount of energy lost from an incident wave due to scattering, per unit volume. Since in single scattering the total energy lost includes the backscattered energy, we must have

$$g(\pi,\omega) \leq g_t(\omega) \tag{7}$$

Further, $g_t(\omega)$ may be calculated from Q and Q_i, by (5) and (6)

$$g_t(\omega) = \omega[1/Q - 1/Q_i]/V \tag{8}$$

If the effect of intrinsic attenuation is ignored, we may calculate an apparent turbidity

$$g_a(\omega) = \omega/[QV] \tag{9}$$

Then,

$$g(\pi,\omega) \leq g_t(\omega) \leq g_a(\omega) \tag{10}$$

Dainty (1981) suggested that $g_t(\omega)$ was constant with frequency between 1 and 30 Hz in the lithosphere.

The final step needed to separate the effects of scattering and anelasticity is a theory that describes the variation of $g_t(\omega)$ and $g(\pi,\omega)$ with frequency, and especially their relationship to each other. So far, I have used an acoustic analog to model the S-wave to S-wave scattering believed to be responsible for the coda, and for the attenuation of the S-wave (Aki, 1980). Clearly, this is too simple a model, and very recently an elastic model has been proposed by Wu and Aki (1984); but for purposes of this report I shall continue to use the acoustic analog.

One result, proposed by Dainty (1981), was that for attenuation for S waves above 1 Hz in the lithosphere, $g_t(\omega)$ was constant with frequency. Dainty used an acoustic model in which the scatterers were considered to be rigid spheres of average radius a, which could be considered to be a scale length describing the size of the scatterers. In this case, $g_t(\omega)$ constant with frequency implies that a is much larger than the wavelength of the waves, or alternatively that we are dealing with the case of high frequency (geometrical) scattering. For this situation both $g_t(\omega)$ and $g(\pi,\omega)$ are constant with frequency and

$$g_t(\omega) = 4\pi \ g(\pi,\omega) \qquad (11)$$

An alternative model used by many investigators to describe scattering is the random medium model, in which the earth is considered to be a medium with constant <u>average</u> values of density and elastic constants, but with random fluctuations of these quantities characterized by an autocorrelation function $N(\vec{r})$, where $\vec{r}$ is the lag. In principle, different quantities (density, elastic constant) can have different autocorrelations, although in a real medium one would expect some connection between the variation of density and elastic constants. If the fluctuations are isotropic, then $N(\vec{r})$ becomes $N(r)$, where the magnitude of the lag vector, r, is considered to be inherently positive. Usually $N(r)$ is characterized by a scale length a such that $N(r) \rightarrow 0$ for $r \gg a$.

Dainty (1983; Appendix 2) considered high frequency scattering for a random acoustic medium with velocity fluctuations only. For this case, there is only backscattering if the autocorrelation $N(r)$ of the fluctuations has the property that

$$\left(\frac{dN}{dr}\right)_{r \rightarrow 0+} \neq 0 \tag{12}$$

If the derivative given in (12) above is also finite, then Dainty (1981) showed that $g_t(\omega)$ and $g(\pi,\omega)$ are constant with frequency at high frequency (wavelength much shorter than a), just as for the sphere. The relationship between $g_t(\omega)$ and $g(\pi,\omega)$ depends on how much forward scattered energy is lost from the wave. Equation (11) is obtained if the treatment of Wu (1982) is followed; only energy scattered at angles of 90° and greater is considered lost. Sato (1982) considered all energy scattered at angles of 30° and greater as lost, as well as a fraction of the energy scattered at angles less than 30°; in this case

$$g_t(\omega) = 56\pi \; g(\pi,\omega) \tag{13}$$

The above discussion assumes that the same fluctuations are responsible for both backscattering and total scattering; if two sets of fluctuations were present, one obeying (12) and the other not, this might not be the case. We would still expect (10) to hold, however.

Another question that must be addressed is the relationship between the results quoted above for an acoustic medium with only velocity fluctuations and the results of Wu and Aki (1984) for an elastic medium with fluctuations of density and both elastic constants. The factors that determine the frequency dependence of the scattered waves seem to be of the same form as those considered by Dainty (1983), suggesting that at high frequencies (wavelengths much shorter than the scale length of the flucuations) $g_t(\omega)$ and $g(\pi,\omega)$ will be constant with frequency as for the acoustic case, provided (12) is met. However, the angular dependence of scattering is in general quite different from the acoustic case, and accordingly (11) or (13) would not hold. Furthermore, different properties of the medium control different types of scattering --for example, if the fluctuations of density and the elastic constants are correlated, Wu and Aki (1984) show that backscattering is controlled by fluctuations of impedance (density times elastic velocity) whereas forward scattering is controlled by velocity fluctuations. Also, mode conversion will occur (scattering from incident P wave into S waves, and vice versa), even at high frequencies if (12) is met, contrary to the results of Knopoff and Hudson (1967).

A final question that must be examined is the effect of multiple scattering. Dainty and Toksöz (1977) and Dainty _et al_. (1974) show that the Q derived from the decay of a coda due to extreme multiple scattering (diffusion approximation) would be Q_i, the anelastic Q; i.e., $Q_s \rightarrow \infty$ in (5) for this case. If we regard less extreme multiple scattering as yielding a Q intermediate between this case and the simple scattering theory, the Q_s will be overestimated and $g_t( )$, from (8) or (9), underestimated if multiple scattering is occurring. It is not clear how the value of $g(\pi,\omega)$ calculated from (4) would be affected; relation (10) might not hold.

Data and Analysis Methods

The earthquakes used in this study occurred in the Mammoth Lakes, California, area. Figure 1 shows the epicenters of the earthquakes and the stations used to located them and for the analysis. For this preliminary effort, two shallow events recorded at three stations each were selected. Relevant data about the events and the stations are given in Table 1. These events were recorded by the United States Geological Survey on digital recorders at 200 samples/second, 12 bit samples. The sensors used were accelerometers, except at station TOM; this should help insure good recording of high frequencies. Only vertical records were used for this study. Further details of the instrumentation and recording methods may be found in Dainty and Duckworth (1983; Appendix 1).

Analysis methods have also been covered in Dainty and Duckworth. Briefly, the power spectrum as a function of time for the whole seismogram is found in a window that is moved along the seismogram at increments of one half the window length. For this study, after some experimentation, a window length of 0.64 sec was chosen. The power spectra were averaged over octave intervals and the coda position fitted to equation (2) by least squares to find the best values of A and Q. About 10 seconds of coda was typically used, and the fit was started at $\sqrt{2}$

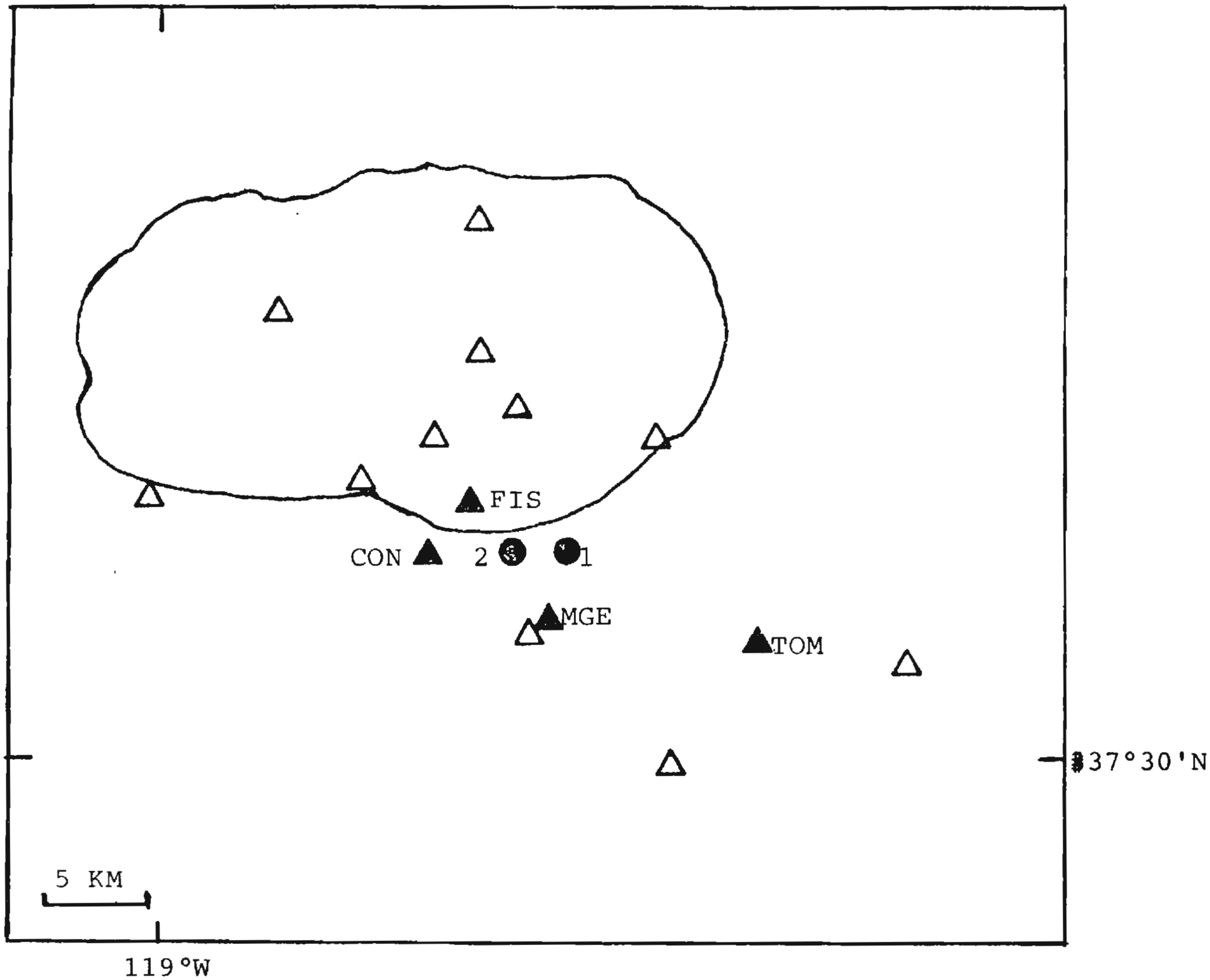

Figure 1. Recording stations (triangles) and epicenters (circles) at Mammoth Lakes, California. Closed triangle - stations used in this study. 1 - event at 148:19:01. 2 - event at 151:07:52.

Table 1. Details of Events and Stations

Event time* /Station	Sensor	S travel time	Range	Latitude	Longitude	Focal Depth
148:19:01				37°35.3'N	118°46.6'W	< 1.0 km
CON	accelerometer	2.81 sec	6.7 km	37°35.38'	118°51.17'	
FIS	accelerometer	2.90	5.5	37°36.84'	118°49.82'	
TOM	velocity	4.24	10.1	37°33.05'	118°40.32'	
151:07:52				37°35.42'	118°48.51'	1.5 km
MGE	accelerometer	1.78	3.7	37°33.67'	118°47.22'	
FIS	accelerometer	2.01	3.3	-	-	
CON	acceleromater	1.81	3.9	-	-	

* Time is given as day:hour:min. Both events occurred in 1980.

times the direct S wave travel time to ensure that only backscattered energy was considered. To estimate the quantity $P_s(\omega)$ in (3), the square of two spectral estimates in the windows nearest the time of the direct S arrival were averaged. This procedure should be valid if the S arrival dominates the signal in the window. Then $g(\pi,\omega)$ was calculated from (4). An S wave velocity of 3.2 km/sec was used.

Results and Discussion

The measurements for $g(\pi,f)$ and Q are presented in Tables 2 and 3. Table 4 shows the values of $g_a(f)$ calculated using (9). Figure 2 plots the values of $g(\pi,f)$ and $g_a(f)$ averaged over all stations and both events against linear frequency f.

Examination of Tables 2 and 4 and Figure 2 reveals that $g_a(\omega)$ is approximately constant with frequency within an order of magnitude. Constant $g_a(\omega)$ would indicate that Q increases linearly with frequency or that coda decay is constant with frequency. This is what would be expected if the intrinsic Q_i were very large, say greater than 5000, and high frequency scattering was the dominant mode of attenuation for a medium where the scatterers are either bodies with distinct contrasts of material properties from the surroundings, or a random medium that meets condition (12), with the derivatives finite (Dainty, 1981; Dainty, 1983, Appendix 2). A random medium that meets (12) is "rough", i.e. it contains discontinuities in the material properties or at least their derivatives. For high frequency scattering, we must have the condition that the scale size of the scatterers is larger than the wavelength at all frequencies under consideration (3 Hz and greater)--this implies a scattering size substantially greater than 200 m.

Table 2. Backscattering Turbidity as a Function of Frequency

Event time	Station	Backscattering turbidity ($km^{-1}steradian^{-1}$) at frequency (Hz)								
		3.0	6.0	9.0	12.0	18.0	24.0	36.0	48.0	72.0
148:19:01	CON	0.52	0.083	0.0081	0.00065	0.00058	0.0011	0.00019	0.00018	0.00062
	FIS	1.6	0.013	0.012	0.0022	0.0017	0.00027	2.3×10^{-5}	2.5×10^{-5}	-
	TOM	1.4	0.62	0.22	0.012	0.0018	0.00072	0.0019	0.0069	0.00048
151:07:52	MGE	0.021	0.006	0.013	0.0046	0.0012	0.00019	6.3×10^{-5}	5.8×10^{-5}	-
	FIS	0.054	0.086	0.067	0.014	0.003	0.0011	0.00031	0.00011	-
	CON	0.046	0.04	0.056	0.010	0.00097	0.00058	0.00047	0.00017	-
Averages		0.6	0.14	0.061	0.0074	0.0015	0.00067	0.0005	0.0012	0.00055

Table 3. Q as a Function of Frequency

Event time	Station	Q at frequency (Hz) 3.0	6.0	9.0	12.0	18.0	24.0	36.0	48.0	72.0
148:19:01	CON	43	110	160	200	310	430	1000	1300	3600
	FIS	43	110	140	1000	125	200	590	1100	-
	TOM	33	83	160	260	430	710	910	1200	1900
151:07:52	MGE	34	170	140	83	91	110	170	380	-
	FIS	43	120	170	140	130	130	200	910	-
	CON	1800	83	63	91	120	130	220	770	-

Table 4. Apparent Total Turbidity as a Function of Frequency

Event time	Station	Apparent total turbidity (km^{-1}) at frequency (Hz)								
		3.0	6.0	9.0	12.0	18.0	24.0	36.0	48.0	72.0
148:19:01	CON	0.14	0.11	0.11	0.12	0.12	0.11	0.07	0.08	0.04
	FIS	0.14	0.11	0.13	0.02	0.29	0.24	0.12	0.08	-
	TOM	0.18	0.14	0.12	0.09	0.08	0.07	0.08	0.08	0.08
151:07:52	MGE	0.17	0.07	0.13	0.29	0.40	0.42	0.42	0.25	-
	FIS	0.14	0.10	0.11	0.17	0.27	0.37	0.36	0.11	-
	CON	(0.0033)	0.14	0.28	0.26	0.30	0.38	0.32	0.12	-
Averages		0.16	0.11	0.15	0.16	0.24	0.27	0.23	0.12	0.06

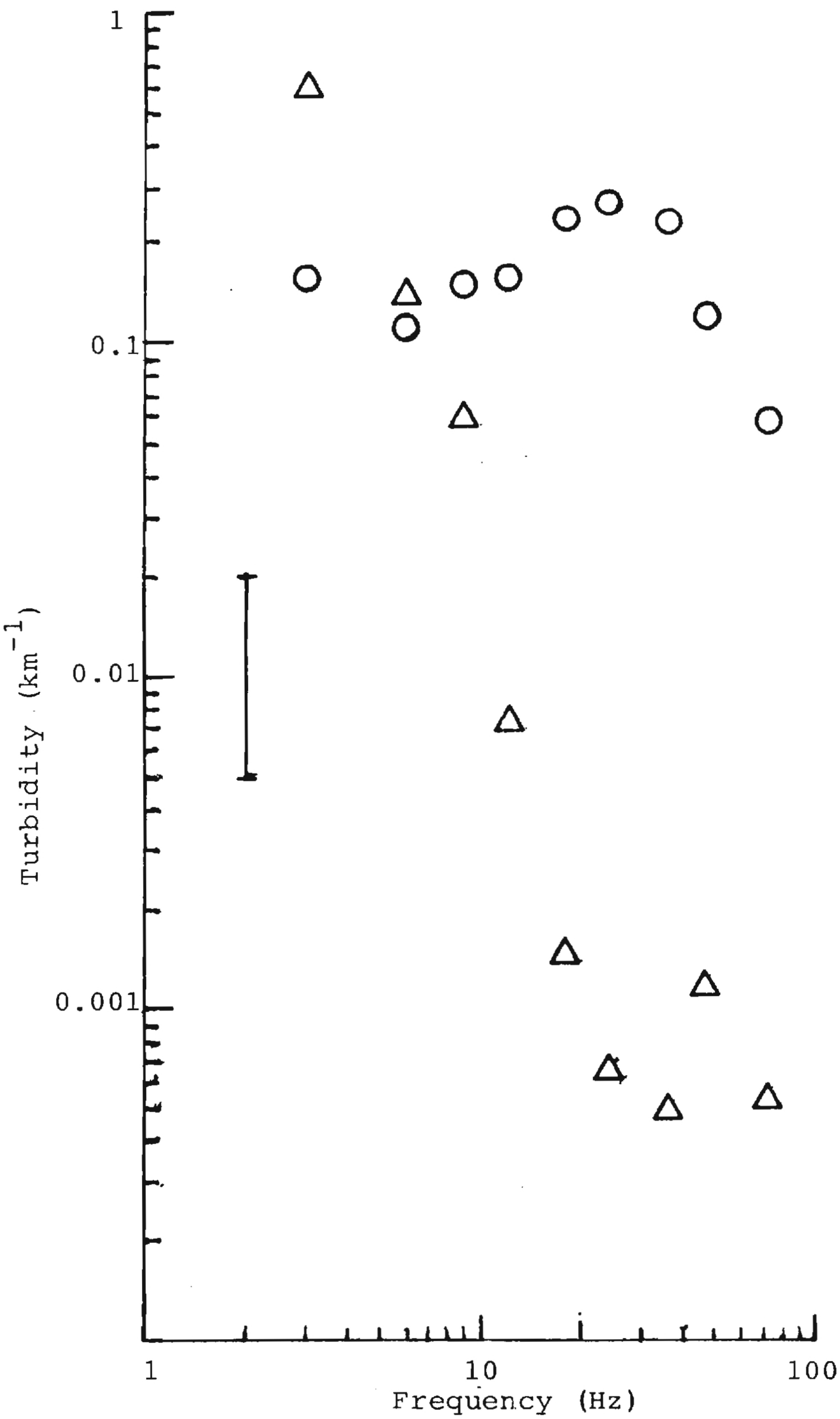

Figure 2. Turbidity as a function of frequency, average results. Triangles - backscattering turbidity. Circles - apparent total turbidity. Error bar indicates a factor of two error, probably typical.

The results for $g_a(\omega)$ discussed above are in approximate accord (but see later discussion) with previous results such as those of Dainty (1981) and Wu (1982) suggesting that single scattering from scatterers large compared to the wavelength is the dominant attenuation mechanism at frequencies of 1 Hz and higher. The results also indicate that $g_t(\omega)$ is approximately equal to $g_a(\omega)$. However, the values of $g(\pi,\omega)$ are not in accord with the single scattering theory of coda formation. First, $g(\pi,\omega)$ is not constant with frequency. In Figure 2, the average value of $g(\pi,\omega)$ at 3 Hz is two to three orders of magnitude larger than values between 18 and 72 Hz. As mentioned previously, $g_a(\omega)$ does not exhibit similar behavior. A similar result is obtained if individual event-station pairs are examined in Table 3: there is a decrease in the value of $g(\pi,\omega)$ of at least two orders of magnitude between 10 and 20 Hz. The theory of single scattering at high frequencies indicates that if $g_a(\omega) \cong g_t(\omega)$ is constant with frequency, then $g(\pi,\omega)$ should be constant with frequency, unless different material properties or different fluctuation populations are responsible for total scattering and backscattering; this point will be discussed below.

Another, even more serious, problem evident in Figure 2 and Table 3 is that $g(\pi,\omega)$ is larger than $g_a(\omega)$ for the event at 148:19:01 at 3 Hz. Similar results were obtained by Andrews (1982). This violates condition (10). Condition (10) follows because in single scattering the energy backscattered is included in the energy lost in attenuation--indeed, relations (11) and (13) suggest that the backscattered energy is only a small part between 1% and 10%, of the energy lost in attenuation.

Before discussing these points, a comparison of $g(\pi,\omega)$ and $g_a(\omega)$ at frequencies of 18 Hz and higher will be made. The average value of $g(\pi,\omega)$ in the range 18 to 72 Hz is 0.00098 km^{-1}/steradian, and the average value of $g_a(\omega)$ is 0.22 km^{-1}. The ratio of these values is $250 \cong 80\pi$. This is the same order of magnitude as (13); if the theory of single scattering is held to apply in this frequency range, this result indicates that most energy lost by scattering in attenuation is forward scattered or side scattered, not backscattered.

The remainder of this discussion is devoted to the apparent discrepancy between the frequency dependence of $g_a(\omega)$ and $g(\pi,\omega)$ in terms of the theories that have been applied to analysis of $g_t(\omega) \cong g_a(\omega)$. One possibility is that the measurement methodology is suspect. This is most serious for $g(\pi,\omega)$, specifically the estimation of $P_s(\omega)$ in (4). Underestimation of $P_s(\omega)$ will lead to values of $g(\pi,\omega)$ that are too large; overestimation of $P_s(\omega)$, to values of $g(\pi,\omega)$ that are too small. The main possibility of error in these cases arises from the window length (0.64 sec). If the S wave pulse length is substantially longer than this, $P_s(\omega)$ will be underestimated ($g(\pi,\omega)$ overestimated). If the S wave pulse length is substantially shorter, and there is a substantial amount of other energy in the window, $P_s(\omega)$ will be overestimated ($g(\pi,\omega)$ underestimated). If one or other of these situations applies equally to all frequencies (i.e., the S wave pulse is non-dispersive), then all values of $g(\pi,\omega)$ will be equally in error: we connot at present exclude this possibility. Aki (1980) argues that the S wave pulse might be dispersive, which would imply different errors in $g(\pi,\omega)$ at different frequencies. However, it seems unlikely that our results can be explained in this way, since to produce a two or three order of magnitude difference between low and high frequencies, the S wave pulse would have to be an order of magnitude longer at low frequencies than at high frequencies. This does not seem reasonable, and is in the opposite sense to Aki's (1980) suggestion (Aki found higher values of $g(\pi,\omega)$ at high frequencies than at low frequencies).

Another possibility is that different properties of the medium, or fluctuations with different autocorrelation functions, control $g_t(\omega)$ and $g(\pi,\omega)$. Wu and Aki (1984) show that if elastic constant and density fluctuations in an elastic medium are correlated, backscattering is mainly due to to impedance fluctuations and forward scattering to velocity fluctuations. Thus it might be argued that the high values of $g(\pi,\omega)$ at low frequencies are due to a set of <u>smooth</u> fluctuations (i.e., (12) is <u>not</u> met) at scale lengths of about 50 m and greater. However, the effect at low frequencies is so large that one would expect to see an effect on $g_a(\omega)$ because of (10). No such effect is seen. Also note that the fluctuations affecting $g(\pi,\omega)$ at low frequencies must be smooth, or they will also produce large values of $g(\pi,\omega)$ at high frequencies (Dainty, 1983; Appendix 2).

So far, the basic assumption of single scattering producing the coda through $g(\pi,\omega)$ and causing attenuation through $g_t(\omega)$ has not been questioned. In view of the difficulties that seem to be inherent in this assumption, consideration of other possibilities seems warranted. One assumption that has been made in the previous discussion is that the Q measured is due primarily to scattering, and that the intrinsic Q_i is very large. This assumption has not been well justified; it is primarily based on the observation that $g_a(\omega)$ is approximately constant with frequency, the "expected" result for high-frequency scattering. Normally, Q_i is considered to be approximately constant with frequency (Knopoff, 1964). If this assumption is allowed and Q is mainly due to Q_i, then $g_a(\omega)$ should increase linearly with frequency. From Table 4 and Figure 2 this does not appear to be true for the averages, although the averages do have the correct behavior between 6 and 24 Hz. Further, examining Table 3, we see that Q is relatively constant between 6 and 24 Hz for the event at 151:07:52, although it is not for the other event. Of course, it is always possible that Q_i is <u>not</u> constant with frequency, but a more fundamental difficulty with the interpretation of $Q \rightarrow Q_i$ with single scattering causing the coda is the large values of $g(\pi,\omega)$ at low frequencies; this certainly implies that Q_s should be an important contributor to Q.

This leaves the final, and perhaps most likely, possibility, namely that multiple scattering is occurring, especially at low frequencies. Theoretical discussion is hampered by the lack of an adequate theory, except in the case of extreme multiple scattering (Dainty and Toksöz, 1977). Here $Q \rightarrow Q_i$ and $g_a(\omega)$ would not bear any relation to $g_t(\omega)$ or $g(\pi,\omega)$. It is not clear what relation $g(\pi,\omega)$ estimated from (2), (3), and (4) would have with physical parameters of the medium. If we consider, for Figure 2, that single scattering is occurring at frequencies higher than 20 Hz, on the basis of the "expected" nature of $g(\pi,\omega)$ and $g_a(\omega)$, then there is probably a set of scatterers of dimension 50 m or greater causing multiple scattering at low frequencies.

The results are preliminary and refer only to the region considered. We hope to present further results for other regions.

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APPENDIX I

OBSERVATIONS OF CODA Q FOR THE CRUST NEAR MAMMOTH LAKES, CALIFORNIA, AND MONTICELLO, SOUTH CAROLINA

By Robert M. Duckworth and Anton M. Dainty

Introduction

The seismic Q of a material is a measure of the degree to which it attenuates the harmonic energy content of seismic waves passing through it. Seismic energy released by earthquakes and underground explosions propagates in the earth primarily in the form of elastic waves. To the extent that earth materials are not perfectly elastic the elastic waves will be attenuated. The energy at an initial time, E_0, and the energy E after elapsed time t are related to the Q by

$$E = E_0 \exp(-2\pi ft/Q) \tag{1}$$

where f is the frequency of the wave. Attenuation is proportional to the distance a wave travels, x, since t = x/B where B is the speed at which the wave propagates. Therefore, knowledge of Q should be useful in predicting the expected amplitudes of seismic waves for yield estimation and other applications. However, because earth materials are inhomogeneous a prediction of seismic wave amplitude is complicated.

Inhomogeneities lead to reflections and refractions, or scattering, of seismic waves. Scattering by these inhomogeneities is an important part of the seismic trace recorded as a function of time. The first part of a seismogram of a local (within a few hundred km of the seismic recording station) earthquake is the Primary or P-wave. This is a compressional wave and travels at speeds up to about 8 km/s for local earthquakes. A second major feature is the Secondary or S-wave which is a shear wave and travels at speeds of about P-speed/$\sqrt{3}$. Between the P and S arrivals and following the S arrival additional waves arrive. Their amplitude decays in a generally exponential manner. Aki (1969) proposed that these waves are the result of scattering of surface waves by surface inhomogeneities. He gave them the name Coda [Latin: tail] waves. Later, Aki and Chouet (1975) considered body wave scattering. Also, scattering affects the attenuation of seismic waves in two ways. Energy is scattered out of the primary wave decreasing the apparent Q. Some of this scattered energy arrives at the receiver in the form of coda waves. The amplitude of these depends upon the scattering mechanism, characterised by the turbidity as described below, and the inelastic Q.

This report presents estimates of the apparent Q from the coda of local earthquakes in two different geologic regions. Attempts are made to separate the effects of scattering and inelasticity in order to determine the turbidity and inelastic Q for each region. The excitation of the coda has also been considered.

Theory

Coda waves are generally considered to be backscattered energy. A model from Aki and Chouet (1975) of coda waves as single, S-wave to

S-wave scatter from a random distribution of scatterers is used in this report.

Observations show the coda to be independent of source-receiver distance, R, for local coda waves (Aki, 1969; Aki and Chouet, 1975; Rautian and Khalturin, 1978). Sato (1977) lends support to this observation with a model for single isotropic scatter. He shows the time dependence of the mean energy density of the coda near the source to be nearly independent of source-receiver distance R for $R << Bt/2$. B is the velocity of S-waves in the medium and t is their travel time measured from the origin time. Because the coda shape is nearly independent of source receiver distance for local coda waves, the model is simplified to the case of coincident source and receiver.

Knopoff and Hudson (1967) modeled scatter from small perturbations in an elastic medium. They found the amplitude for S to S scatter to dominate that of P to P scatter by $(\alpha/B)^2$ for geometric (high frequency) and $(\alpha/B)^4$ for Rayleigh (low frequency) scatter and P to S and S to P scatter to be "vanishingly small" for geometric scatter. α and B represent the speeds of P and S waves respectively. For the crust $\alpha = \sqrt{3}\,B$ is a good approximation and the amplitude of S to S scatter is greater than P to P scatter by a factor of 3 for geometric scatter. The observed character of coda waves also supports S to S dominance (Aki, 1980).

Before presenting a model of coda waves the effects of attenuation will be considered. Plane waves traveling in a medium with a random distribution of scatterers will be attenuated. Let E_0 represent the average energy of the waves before scattering and E represent the energy of plane waves traveling in the same direction as the primary waves after scattering. We assume the primary cause of attenuation by scattering will be backscatter. For $(E_o-E) << E_o$ over a distance equal to the average separation of "backscatterers", the attenuation of primary waves is given by

$$E/E_o = \exp(-x\rho\sigma_b). \tag{2}$$

The backscatter cross section σ_b represents the average of the product of the backscatter function and the projected area of the scatterers, or the average apparent area of the scatterers. ρ is the number of scatterers per unit volume. As σ_b and ρ are unknown for the crust, their product is often replaced by a scattering coefficient, g, called the turbidity. The turbidity represents the fractional loss of energy due to scatter per unit distance traveled. The term was introduced by Chernov (1960) and first used in seismic literature by Galkin et al. (1970). They employ it to describe their observed high frequency spatial variation of seismic amplitudes as a function of distance from the source. Equation (2) may also be applied to the attenuation a plane wave suffers when passing through a medium where scattering is occurring as a result of random fluctuations in the medium properties rather than discrete "backscatterers". In this case, g is a function of the statistical properties of the medium and the wavelength (Wu, 1982).

In the earth medium there will also be attenuation resulting from inelasticity. This is attenuation that occurs as elastic energy is converted to heat. For $(E_o-E) << E_o$ over a wavelength, the inelastic attenuation is given by

$$E/E_0 = \exp(-2\pi fx/BQ). \tag{3}$$

Q is the quality factor, B is the speed of shear waves, x is the distance they travel in the medium and f is frequency. The inelastic attenuation mechanism is not well understood. It is generally observed that Q is independent of frequency over the range of seismic frequencies studied. This is in agreement with the results of Knopoff (1964) who studied Q in the laboratory and found it independent of frequency for dry materials. These two attenuation effects can be combined. In terms of the time the energy spends in the medium $t = x/B$ and for constant B,

$$E/E_0 = \exp(-2\pi ft/Q) \;.\; \exp(-Btg) \tag{4a}$$

$$= \exp[-t(2\pi f/Q_T)]. \tag{4b}$$

Q_T represents the combined effects of scattering and inelastic attenuation.

$$1/Q_T = 1/Q_i + gB/2\pi f \;, \tag{5}$$

where Q_i is the inelastic Q (Dainty, 1981). Dainty and Toksoz (1981) suggest that for a single scatter theory to apply $2\pi f/Q_i >> Bg$ is a necessary condition, as otherwise there would be sufficient energy for higher order scattering.

The power spectrum of the coda as a function of time under the conditions discussed above has been derived by Aki and Chouet (1975). If the spectrum of the source is $\phi(f|r_0)$ at a reference distance r_0

from the source, the coda power spectrum is

$$P(f|t) = st^{-2m} \exp(-2\pi ft/Q_T) \tag{6}$$

where

$$s = |\phi(f|r_0)|^2 \; 4m\pi r_0^{2m}(B/2)^{1-2m} \cdot g \tag{7}$$

s represents the source term, the turbidity $g = \rho\sigma_b$, and $m = 0.5$ for cylindrical (surface wave) or $m = 1.0$ for spherical (body wave) spreading. The source term, s, will in general depend on the scatterers through its dependence on g. The time t is measured from the origin time of the event.

The average coda amplitude at a particular frequency and time $A(f|t)$ is related to the power spectrum.

$$A(f|t) = [P(f|t)\Delta f]^{1/2} \tag{8}$$

Δf is the band width considered about f. Q_T can therefore be measured from the coda wave amplitude attenuation using equation (6). Sato (1977) points out the single scatter model is only applicable for a coda duration $t < 1/gB$. Longer times imply a significant contribution of higher order scattering which effects the return of scattered energy to the later parts of the coda. The decomposition of Q_T to find Q_i and g depends upon assumptions about their frequency dependence. This and the applicability of the single scatter model will be considered in the section on results.

Data

Two sets of earthquake data are available for analysis courtesy of Paul Spudich (U.S. Geological Survey at Menlo Park, California). Both sets contain digital records of earthquakes and their locations. Program Hypoinverse by Klein (1978) is the method of location. Locations and primary S-wave travel times of the earthquakes selected for analysis have been read from the U.S.G.S. tapes (Tables 1 and 2). The first set is from a series of events associated with the filling of Monticello Reservoir in South Carolina. It contains records of 32 earthquakes, recorded in May and June of 1979 and used in a study of stress drops by Fletcher (1982). He reports they are all magnitude $M < 1.7$. Secor <u>et al</u>. (1982) report the earthquakes as occurring in a heterogeneous quartz monzonite pluton of Carboniferous age. Zoback and Hickman (1982) suggest that the earthquakes are shallow, occurring along existing fractures, and the result of an increase in pore pressure caused by increasing the head at the reservoir. Figure 1 adapted from Talwani and Hutchenson (1982) shows the location of the seismic stations and the reservoirs. Epicenters for the earthquakes considered have been indicated on this figure. They are all within 8 km of the recording stations. Secor <u>et al</u>. (1982) report on the two reservoirs. Parr Shoals reservoir was impounded in 1914 and increased in 1976 to a volume of 0.039 km^3 with a surface area of 18 km^2 and a maximum depth of 11 meters. Monticello reservoir has a volume of 0.5 km^3, a surface area of 27 km^2, and a maximum depth of 48 meters.

Fletcher (1982) describes the instrumentation used to record the Monticello earthquakes. A summary is given here. Sprengnether

Table 1. Locations of earthquakes and recording stations at Monticello, South Carolina.

EVENT/STATION	"S" TIME	RANGE	LOCATION
1271000			34 20.03 N 081 19.62 W
DUC	0.68	2.2	34 20.07 N 081 21.06 W
DON	1.05	3.5	34 21.42 N 081 21.20 W
LKS	0.89	3.0	34 19.95 N 081 17.69 W
1320218			NOT POSSIBLE
JAB	1.66	5.8	34 22.28 N 081 19.47 W
1281119			34 20.70 N 081 20.81 W
JAB	1.31	4.6	
1302328			34 20.49 N 081 20.65 W
SNK	0.56	1.8	34 20.29 N 081 19.54 W
JAB	1.19	3.8	
LKS	1.38	4.7	
1310603			34 18.50 N 081 20.50 W
SNK	1.05	3.7	
LKS	1.48	5.2	
JAB	2.00	7.0	
1281831			NOT POSSIBLE
JAB	1.02	3.57	

Table 2. Locations of Earthquakes and Recording Stations at Mammoth Lakes, California.

EVENT/STATION	"S" TIME	RANGE	LOCATION
1571941			37 32.91 N 118 52.53 W
MGE	3.17	8.0	37 33.67 N 118 47.22 W
HCF	4.37	10.6	37 38.51 N 118 50.98 W
CBR	5.50	15.2	37 40.75 N 118 49.51 W
TWL	4.84	13.7	37 36.93 N 119 00.37 W
ROC	5.67	14.9	37 29.78 N 118 43.16 W
TOM	6.11	18.0	37 33.05 N 118 40.32 W
LKM	5.97	17.3	37 41.80 N 118 56.13 W
1592317			37 37.50 N 118 52.52 W
HCF	2.38	3.0	
FIS	2.58	4.2	37 36.84 N 118 49.82 W
CBR	3.60	7.5	
LKM	3.73	9.6	
MGE	4.03	10.6	
TWL	4.52	11.6	
LAK	5.30	13.1	37 38.49 N 118 43.70 W
TOM	6.78	19.7	
ROC	6.71	19.8	

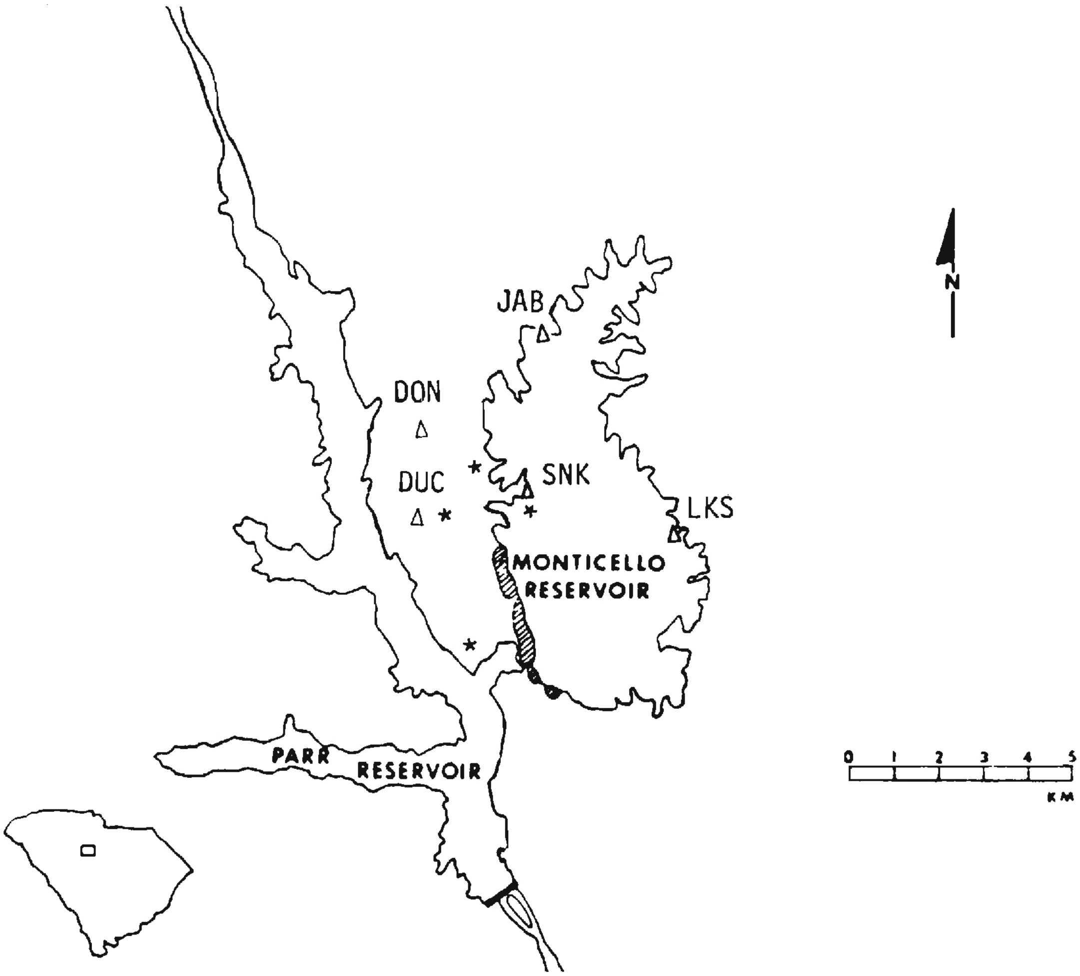


Figure 1. Recording Stations and Epicenters at Monticello Reservoir, South Carolina. stations(Δ), epicenters(✱)

DR-100, 12-bit, digital recorders were used with an unspecified make of geophone. The sample rate was 200 Hz with an antialiasing filter of either 50 Hz or 70 Hz having a 30 db per octave rolloff. The geophones were velocity sensors with damping specified as 0.7 of critical. The possibility of aliasing on Monticello records for which the filter corner frequency was set at 70 Hz was considered. Inspection of bandpass filtered records at the higher frequencies revealed no difference in the rate of amplitude decay between the recordings made with a 50 Hz corner filter and a 70 Hz corner filter.

The second set of data is from Mammoth Lakes, California. It consists of the 150 earthquakes used by Archuleta et al. (1982) in a study of source parameters. The earthquakes followed a series of magnitude M > 6.0 earthquakes which occurred May 25, 1980 (Spudich et al., 1981). Wallace et al. (1982) investigate a discrepancy in fault plane projections as reported in the literature for the Mammoth Lakes earthquakes. Citing evidence of an active magma body, they suggest a low velocity zone related to recent magmatic activity as a possible cause. Figure 2 adapted from Spudich et al. (1981) shows the station locations. Epicenters for the earthquakes considered have been added to this figure. The source receiver distances range from 5 to 25 km. The Long Valley Caldera, a volcanic crater 0.7 million years old, is outlined along with major faults. The instrumentation at Mammoth Lakes is essentially the same as at Monticello, except that accelerometers are used at some of the Mammoth stations. Details of the instrumental operations and clock corrections can be found in Spudich et al. (1981).

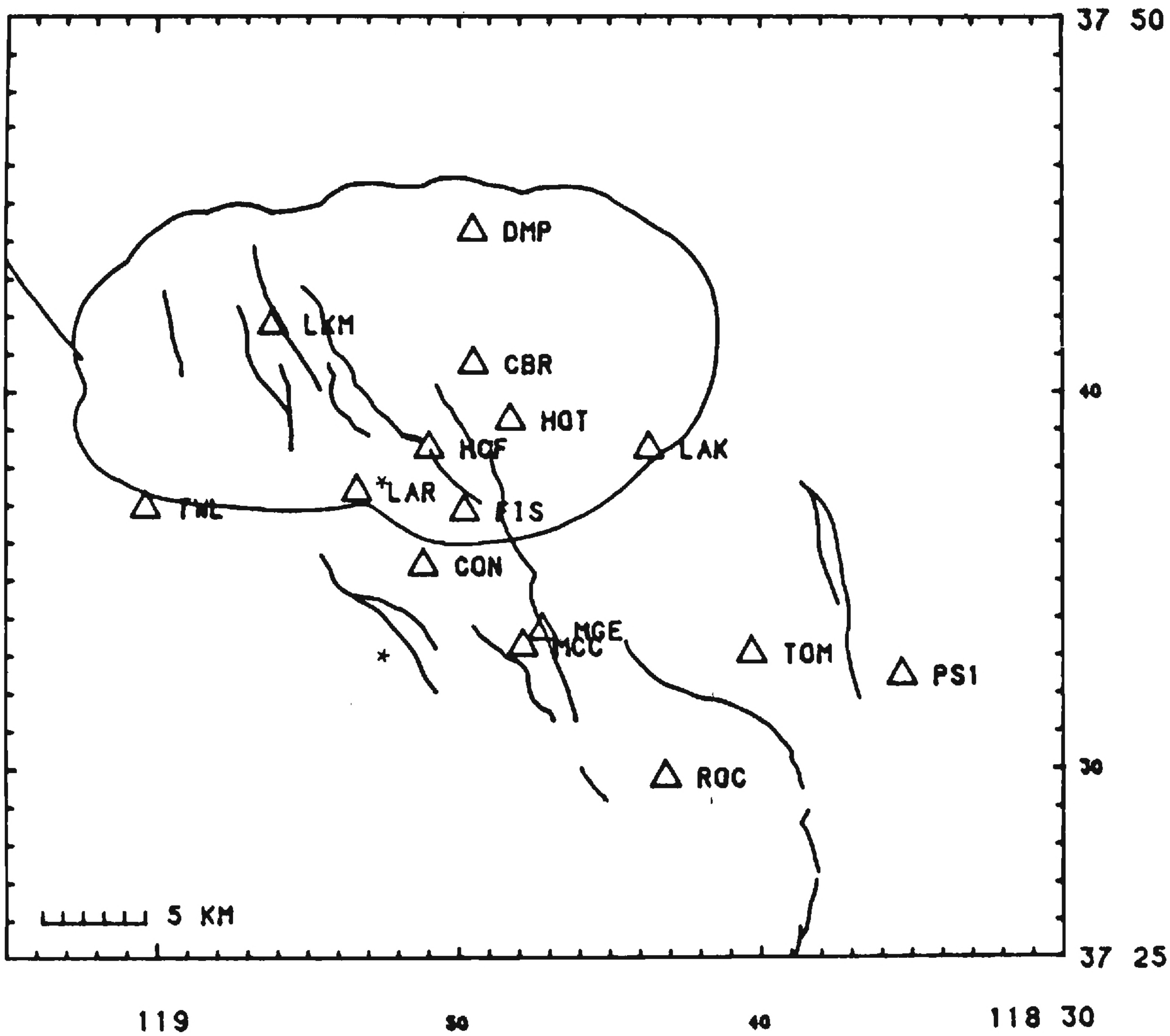


Figure 2. Recording Stations and Epicenters at Mammoth Lakes, California. stations(Δ), epicenters(*)

Analysis

The rms coda amplitude is related to the power spectrum by $A(f|t) = [P(f|t)\Delta f]^{1/2}$. Substituting the equation for the power spectrum in the single scatter model gives

$$A(f|t) = ct^{-m} \exp(-\pi ft/Q_T) \ . \tag{9}$$

Over the range of frequencies for which geometric scatter is the case, σ_b is constant. Therefore, $c = (s\Delta f)^{1/2}$ has the same frequency dependence as the source for octave Δf. For cylindrical waves $m = 0.5$, while for spherical waves $m = 1.0$. Taking the ordinary logarithm of this expression

$$\log[A(f|t)] = C - m \cdot \log(t) - bt \tag{10}$$

is obtained where $b = \log(e)\pi f/Q_T$ and $C = \log(c)$. To estimate Q_T this equation is fit to filtered records by the method of least squares. This is the approach taken by Aki and Chouet (1975).

To study the temporal decay of coda waves the longest records are chosen. Of these the duration of recording ranges from 8.0 to 23.0 seconds at Monticello and from 10.0 to 35.0 seconds at Mammoth Lakes. Only a few records from Monticello are considered to be of sufficient duration for coda analysis (Table 1). The Mammoth Lakes records chosen for analysis are listed in Table 2.

Travel times are taken from the location data provided with the U.S.G.S. tapes. Some of the Monticello earthquakes were not located. For these the S-wave travel time is calculated from the difference in S and P wave arrival times and assumed S and P wave velocities of

3.5 km/s and 6.0 km/s respectively. Ground displacement amplitude as a function of time in octave wide bands is determined for the selected records by the following procedure.

The mean is removed from a record in a 256 sample (1.28 second) window centered about the origin time. This portion of the trace is multiplied by normalized Hamming coefficients. The windowed trace is corrected for instrument gain, Fourier transformed, and multiplied by $1/f$ or $1/f^2$. Assuming a flat instrument response in the frequency band of interest, this converts the trace to ground displacement amplitude from velocity and acceleration records respectively. It has been assumed, as in Archuleta _et al_. (1982) and Fletcher (1982), that the velocity response is flat from 2 Hz to 50 Hz and the acceleration response is flat above 0.1 Hz. The average amplitude is calculated within octaves (equal power bands) about selected center frequencies. The log of this amplitude is plotted at time zero on separate plots for each frequency. The window is then moved 100 samples (0.5 seconds) and the process is repeated with the log average amplitude plotted at time 0.5. This is done until the end of the record is reached. The result is a series of plots of log average amplitude versus time, measured from origin time, at selected center frequencies from 3 Hz to 72 Hz. Figure 3 contains a sample record and its corresponding filtered amplitude plots. Frequencies above the corner of the antialiasing filter are considered as the constant reduction in amplitude will only show up in the source term. Therefore Q_T can still be estimated provided there is sufficient energy present at these frequencies.

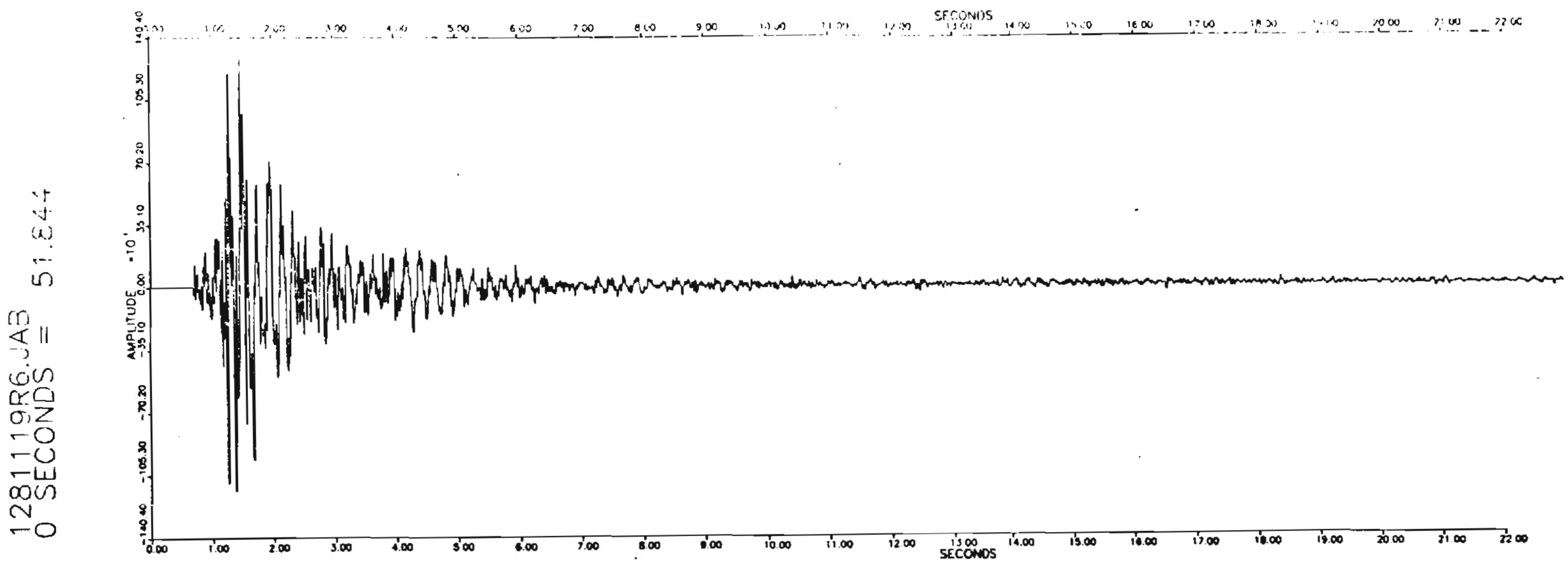
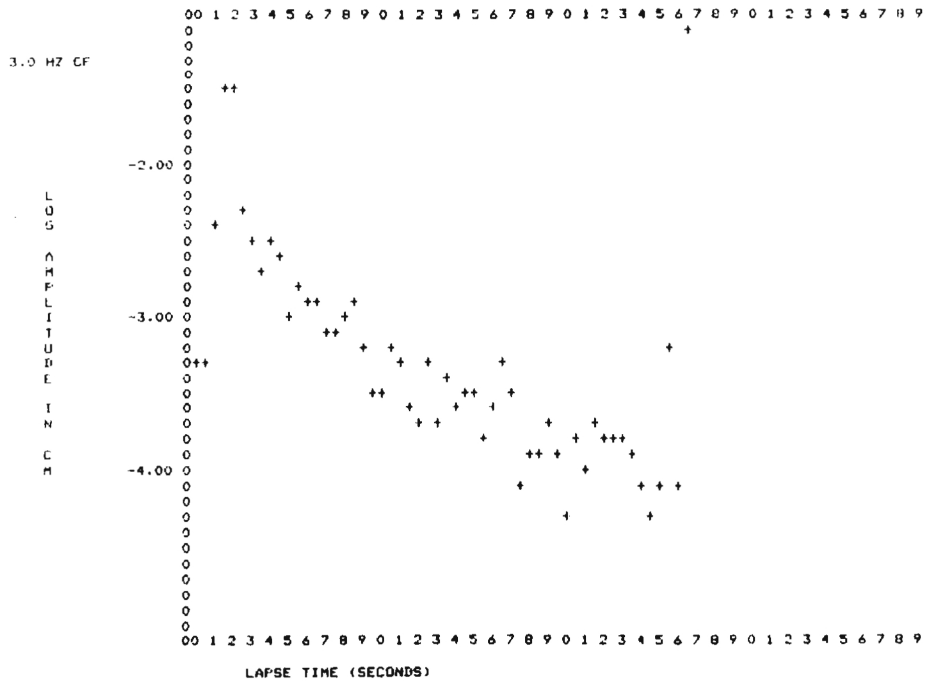


Figure 3a. Record of Event 1281119 at Monticello Reservoir. The amplitude scale is digitizing units.

Figures 3b-h. Log(amplitude) versus Time for Event 1281119at Monticello. (following 4 pages) Plots b-j correspond to center frequencies of 3, 6, 9, 12, 18, 24, 36, 48, and 72 Hz respectively. It can be seen from the coda that Q is changing with time. The early coda amplitude falls off rapidly while the late coda shows little decay.



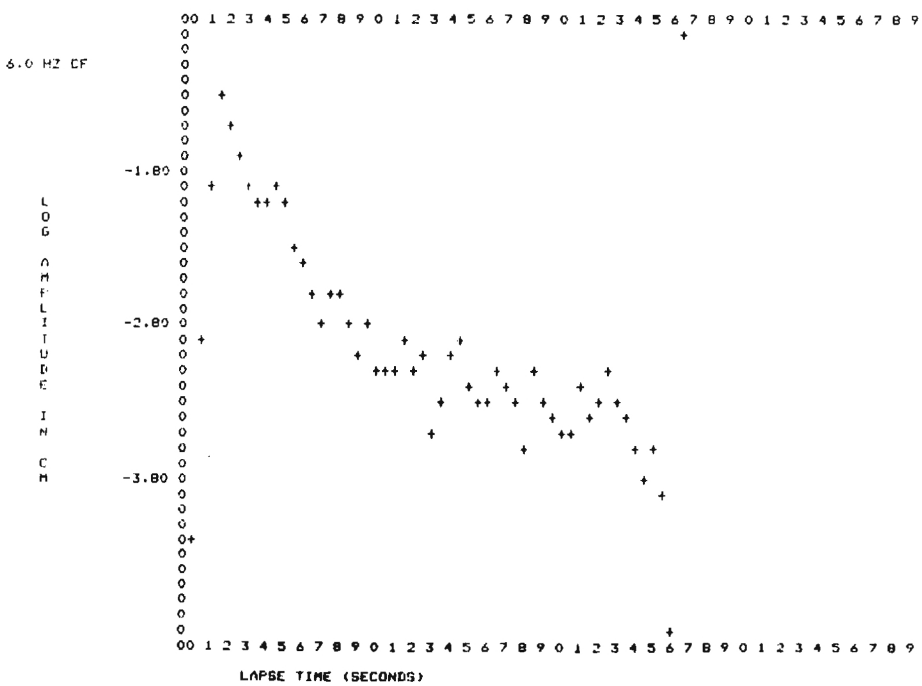
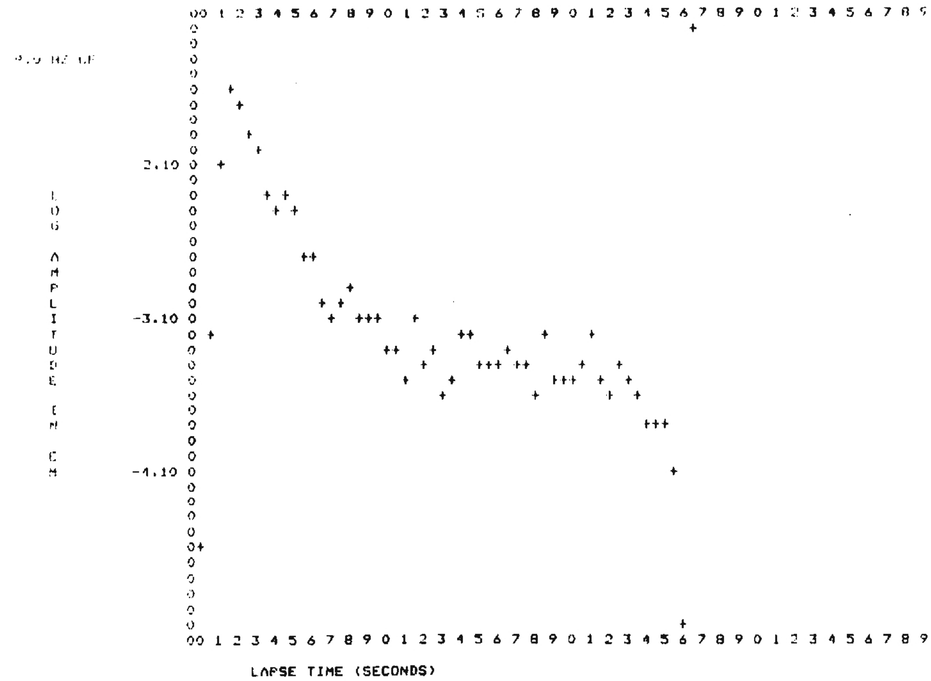


Figure 3 . b.(top) c.(bottom)



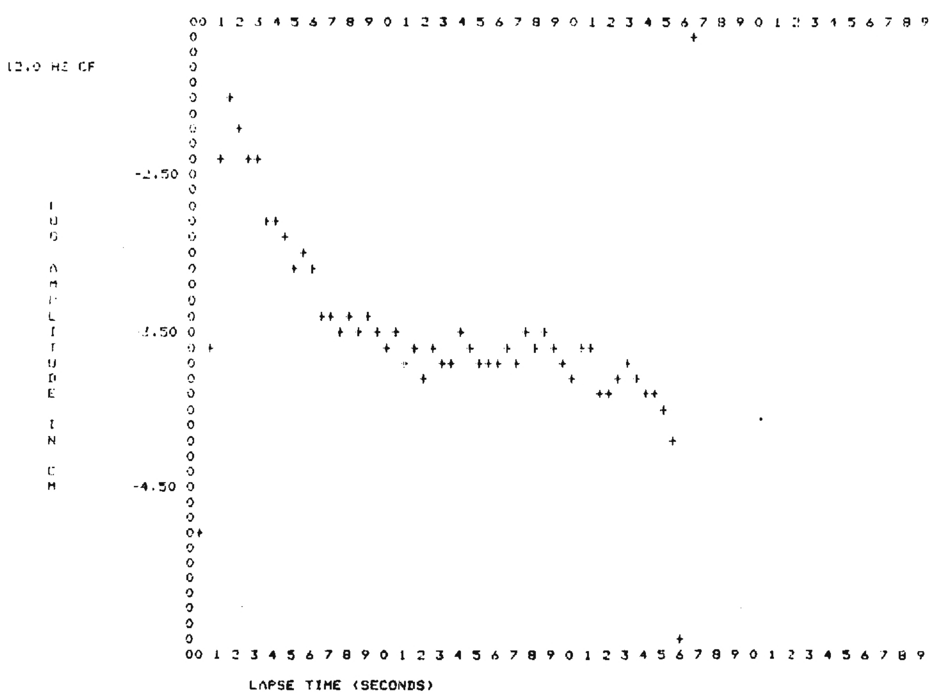
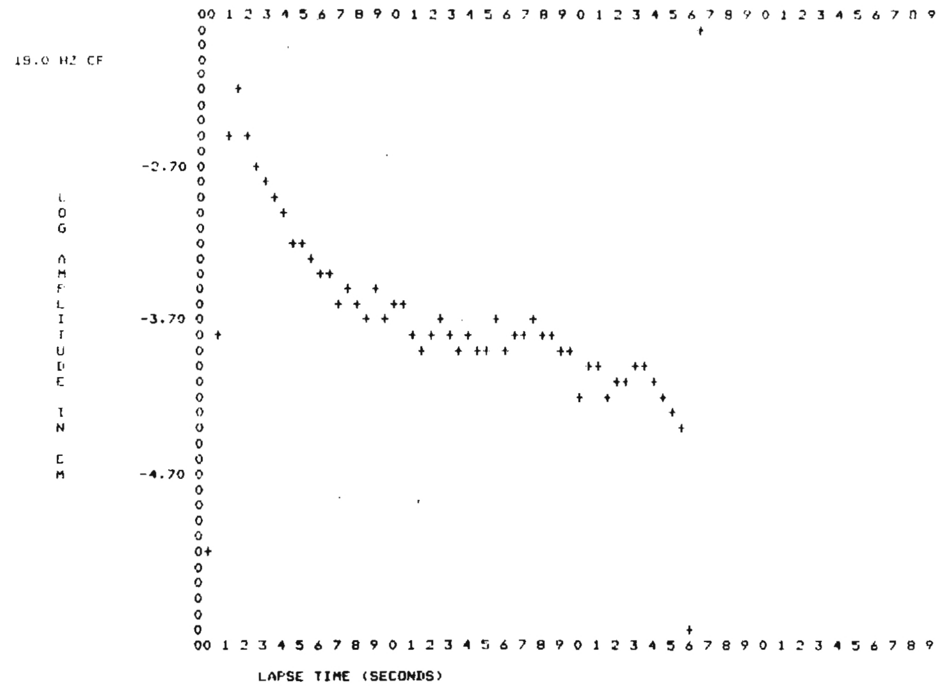


Figure 3. d.(top) e.(bottom)



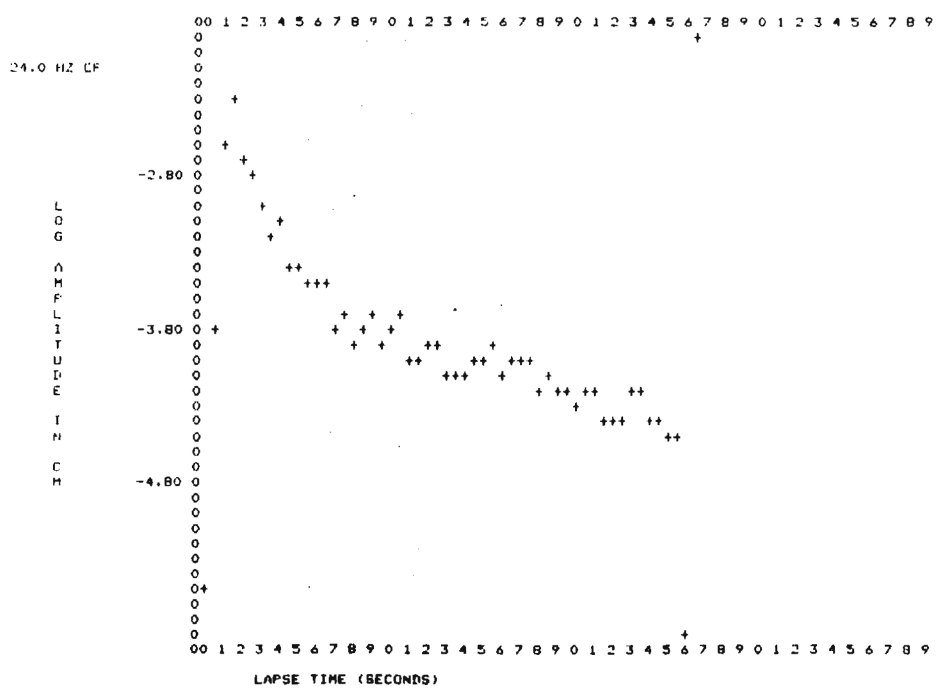
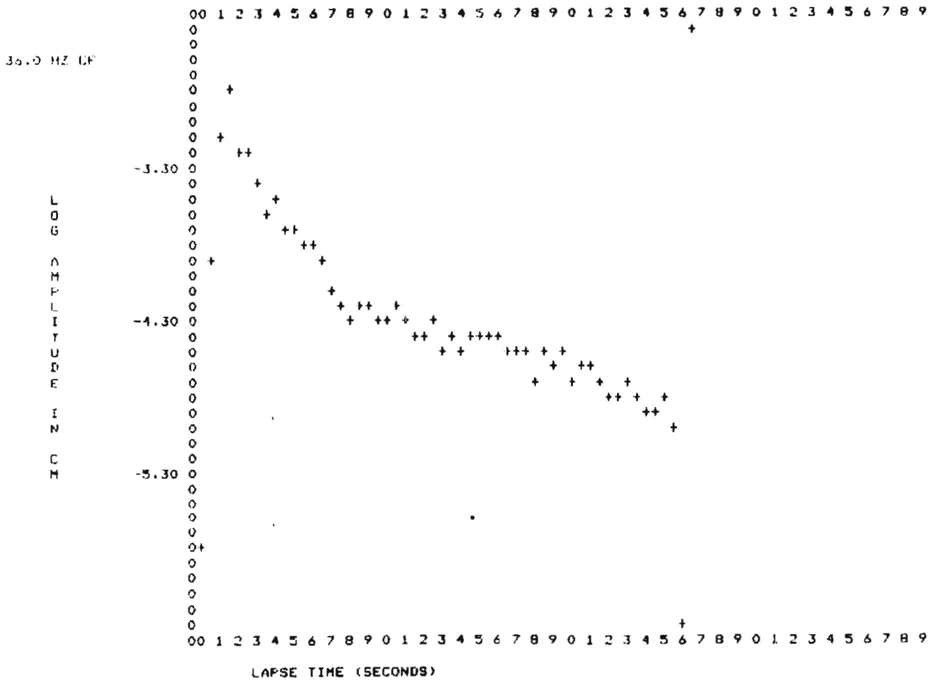


Figure 3. f.(top) g.(bottom)



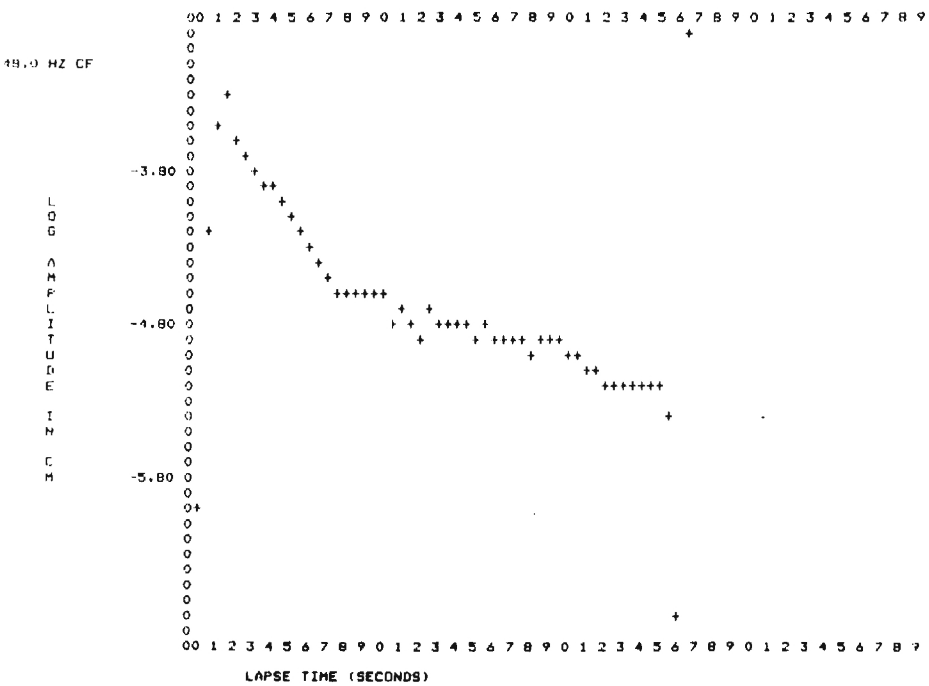


Figure 3. h.(top) i.(bottom)

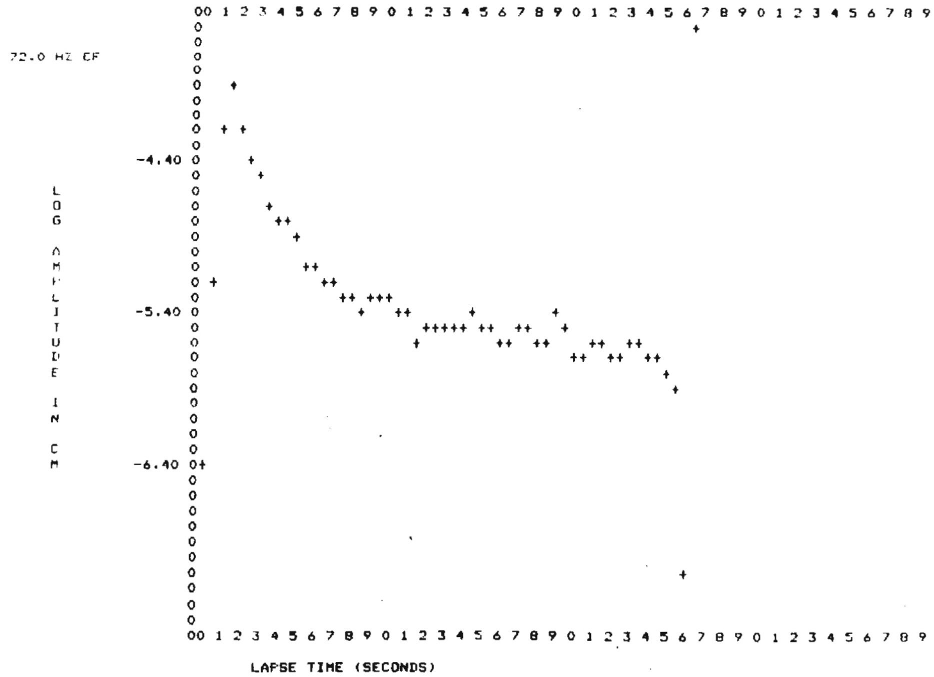


Figure 3. j.

The next step is to fit the model equation for $\log[A(f|t)]$ to the backscatter portion of the $\log[A(f|t)]$ plots for an estimate of Q_T. The method of least squares is used and two fits are made corresponding to the two choices of m = 1.0 and m = 0.5. Standard deviations are also calculated for each fit. Attempts to fit the model taking m as an independent variable produced erratic results as has been reported in the literature (Aki and Chouet, 1975).

The backscatter portion of the filtered data is determined as follows. Energy arriving after $\sqrt{2}$ times the primary S-wave travel time is backscatter in the sense that the scattering angle is greater than 90 degrees. However, because of the 1.28 second window and 0.5 second steps used for averaging only arrivals 1.0 second later than this are entirely backscatter.

On some records the dynamic range of the recording instrument has been exceeded for the early coda. The saturated portion of the coda is not considered in the analysis. Also excluded are the last three points of the filter output as they are distorted by artificial extension of the end of the trace necessary in filtering.

Another consideration is noise. The background noise level is assumed constant and taken equal to the peak noise level recorded prior to the P-wave arrival. If no noise is recorded here it is assumed to be + or - 0.5 times one unit of digitization. The noise level is converted to ground displacement amplitude. A reference, five times (14db above) this noise level, is taken as the minimum acceptable signal level. Portions of the coda for which the average signal amplitude falls below this reference are not considered in the fits.

Finally, from the longest records it is evident that Q_T changes with time causing the Q_T value obtained to depend on the portion of coda fit. This can be seen on the sample record in Figure 3. This time dependence of Q_T was also observed by Rautian and Khalturin (1978) who suggest variations of Q with depth and temporal variations of the geometric spreading factor as possible causes. The variation in spreading factor could result from a change in the type of waves being recorded as a function of time (e.g., surface to body waves as time increases). Another possibility is that of a significant deterministic influence on the energy arriving at the receiver. Long and Wilson (1982) have noticed resonance peaks in the power spectra of the Monticello records. The frequencies at which these occur are in congruence with conceivably supportable resonances in the Monticello reservoir. This can be regarded as an example of multiple scattering, which tends to raise the value of Q because energy previously scattered out of the coda is scattered back in, reducing the rate of decay. Indeed, in the limit of strong scattering, there is no contribution to Q from scattering (Dainty and Toksöz, 1977). Because of the observed temporal dependence of Q_T, fits are made to different portions of the coda where possible.

Results

Plots of the average over all stations and events of log[Q_T] versus log[frequency] are constructed and appear as Figure 4. Because a temporal dependence of Q_T was observed, three plots are made for each area and m value. They are from the beginning of backscatter to: I, five seconds beyond; II between five and nine seconds beyond; III,

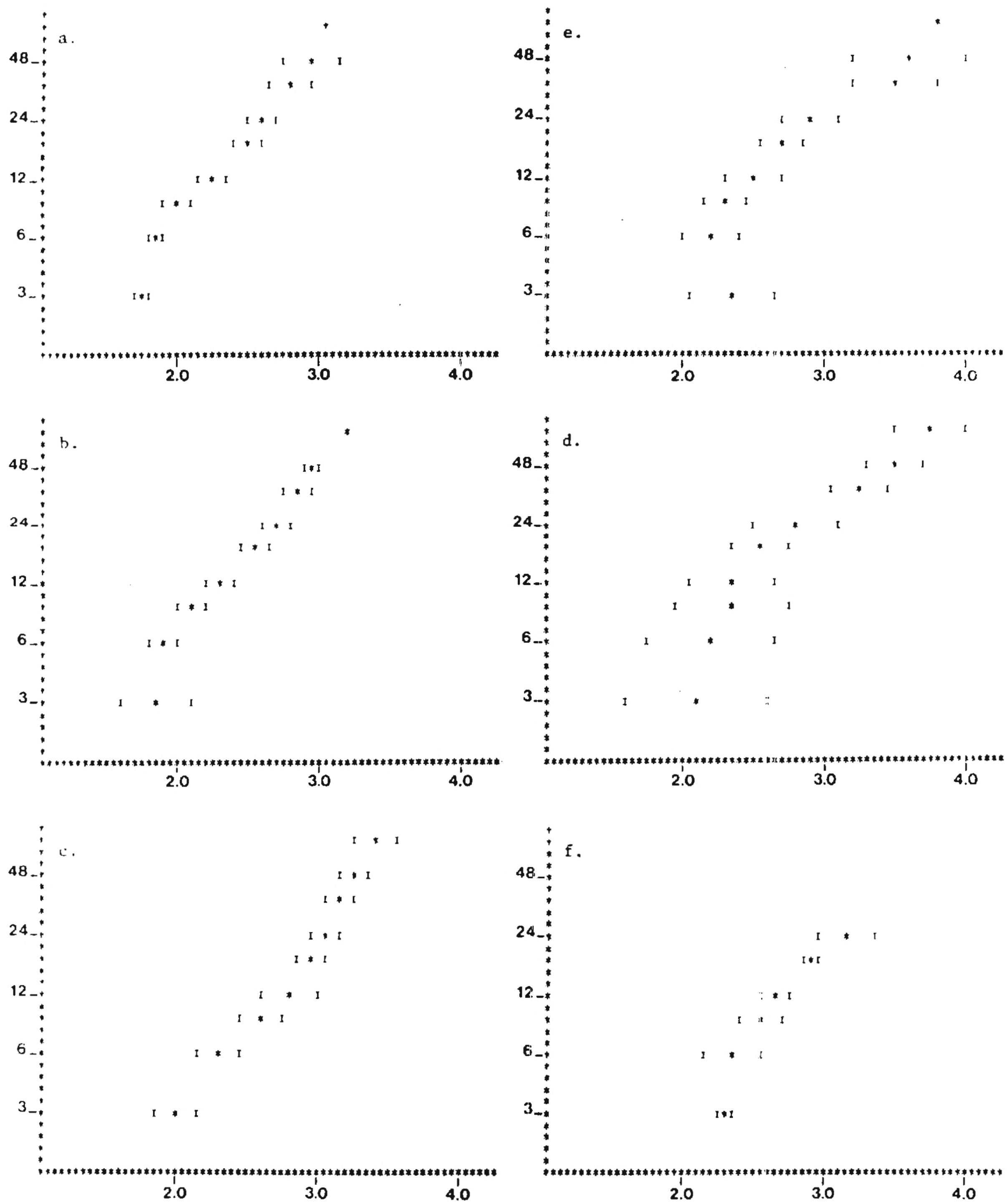


Figure 4a-f. Average log(Qtotal) versus Frequency, m=0.5. The vertical axes are frequency and the horizontal axes are log(Qtotal)
a, b, and c are for t< 5, 5< t< 9, 9< t respectively at Monticello.
d, e, and f are for t< 5, 5< t< 9, 9< t respectively at Mammoth.

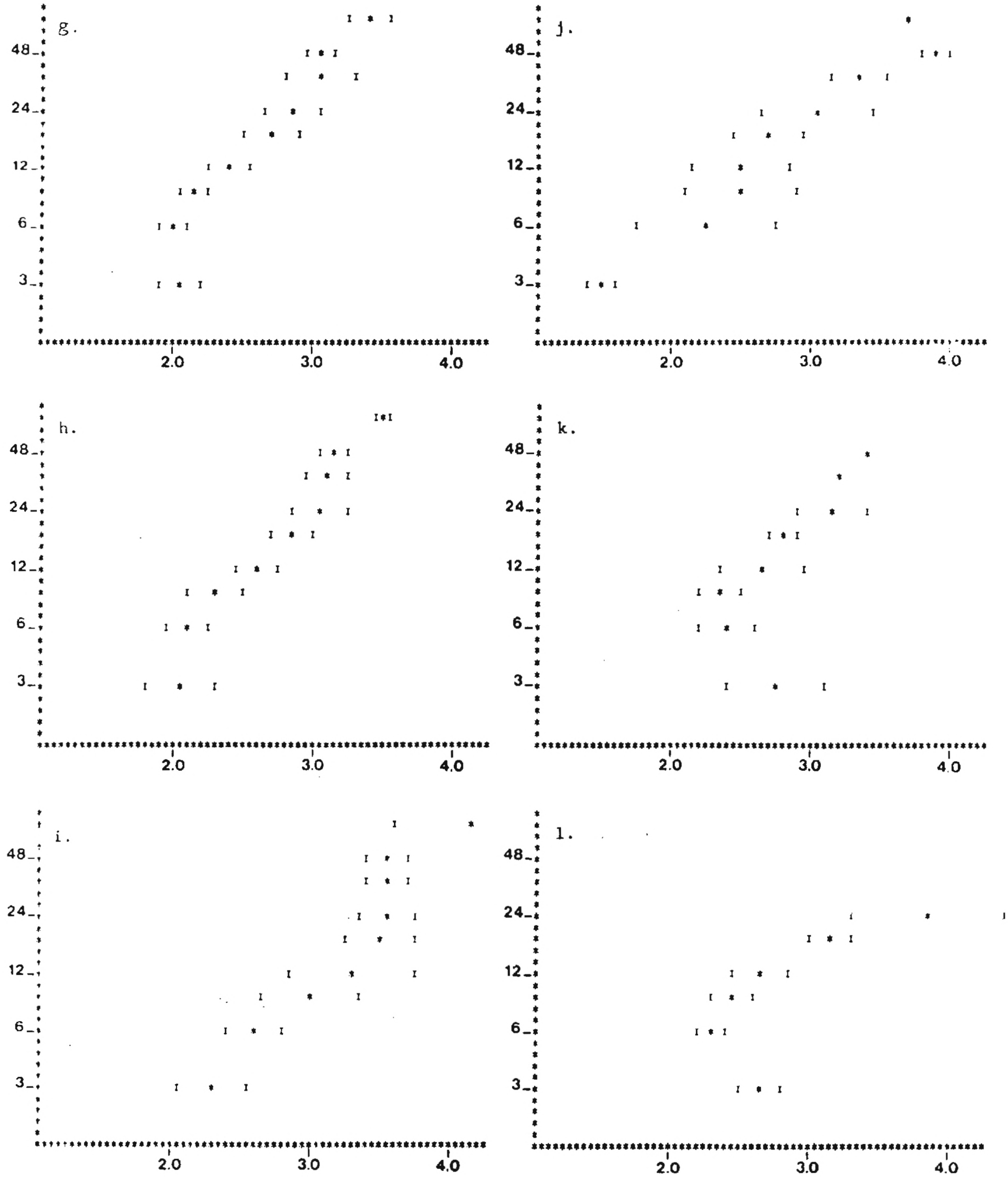


Figure 4g-1. Average log(Qtotal) versus Frequency, m=1.0. The vertical axes are frequency and the horizontal axes are log(Qtotal)
a, b, and c are for t< 5, 5< t< 9, 9< t respectively at Monticello.
d, e, and f are for t< 5, 5< t< 9, 9< t respectively at Mammoth.

more than nine seconds beyond. These times were chosen arbitrarily within the limits of the data. As the plots are roughly linear, best fits of the equation $Q_T = af^b$ are made to each of these to determine the values of a and b. This approach is common in the literature. The a and b values obtained are presented in Table 3.

The frequency dependence of Q_T may be used to estimate Q_i and g if some assumptions are made about their frequency dependence. For the single backscatter model applied here the density of scatterers ρ is assumed constant. The backscatter cross section σ_b will be a constant for geometric (high frequency) scatter. The result is a constant g for geometric scatter with the single backscatter model. It is usually assumed that Q_i is independent of frequency in agreement with most laboratory measurements of Q_i (Knopoff, 1964). With these assumptions a least squares fit of $1/Q_T = 1/Q_i + gB/2\pi f$ is made to the Q_T versus frequency data to estimate g and Q_i (Dainty, 1981). Negative values of Q_i occur when the least square fit results in a $gB/2\pi f$ which is greater than $1/Q_T$. As Q_i increases with increasing $gB/2\pi f$ a negative Q_i result is indicative of a very high Q_i value. These results are shown in Table 4.

A second estimate of g is obtained as follows. From the least squares fit of log[A(f t)] estimates of Q_T and the source factor have been obtained. The filter output at the primary S-wave travel time at a given frequency is a component of the source factor C. This allows a second estimate of g from the primary S-wave spectra and the source factor C = log (sΔf).

Taking a result from the theory section

Table 3. Best Fit a and b Values. $Q_T = af^b$ at Monticello and Mammoth. m = 0.5 and m = 1.0 correspond to cylindrical and spherical spreading respectively.

	Monticello			Mammoth		
	a	b	freq	a	b	freq
<u>m = 0.5</u>						
I	6.3	1.2	6-48	1.5	1.8	3-24
I				6.2	0.6	12-72
II	9.7	1.1	3-72	4.1	1.6	6-72
III	31.8	1.0	3-72	16.6	1.3	6-24
<u>m = 1.0</u>						
I	9.2	1.2	6-72	9.0	1.4	3-36
II	11.0	1.2	6-72	20.0	1.2	6-48
III	29.6	1.5	3-24	7.6	1.7	6-24
III	2750	0	18-48			

Table 4. Log(g) Values From Fitting Equation (5).

	Monticello			Mammoth		
	log g	Q_i	freq	log g	Q_i	freq
<u>m = 0.5</u>						
I	-0.92	940	3-72	-1.28	1050	3-72
II	-1.00	900	3-72	-1.46	860	3-72
III	-1.19	*	3-72	-1.49	1050	3-24
<u>m = 1.0</u>						
I	-0.70	*	3-72	-1.17	1000	3-72
II	-1.92	610	3-72	-1.17	2960	3-48
III	-1.49	*	3-72	-1.82	580	3-24

* indicates Q_i approaching infinity.

$$s = \left|\phi\,(f|r_o)\right|^2 \, 4m\pi r_o^{\,2m}\,(B/2)^{1-2m}\,g\ . \qquad (11)$$

Also, the average amplitude, $|\phi\,(f|r)|$, of waves incident at a scatterer at distance r is

$$|\phi(f|r)| = |\phi(f|r_o)|\,(r/r_o)^{-m}\,\exp(-\pi fr/BQ_T)\ . \qquad (12)$$

It is important to note that $|\phi\,(f|r)|$ is also the average spectral amplitude of the direct S-wave where r is the source receiver distance. This is estimated from a linear interpolation of the spectral amplitudes measured in the two windows closest to the S arrival time. The interpolation is necessary as the window is generally not centered on the S arrival. Solving equation (12) for the source spectrum at r_o, $|\phi\,(f|r_o)|$, and substituting in (11) a solution for log(g) is obtained.

$$\log(g) = 2C - \log[4m\pi(B/2)^{1-2m}\,|\phi(f|r)|^2\,r^{2m}\,\exp(2\pi ft/Q_T)\Delta f\ ]\ . \qquad (13)$$

Estimates of average log(g) are obtained using this equation for m = 0.5 and or m = 1 and are tabulated in Table 5.

<u>Discussion</u>

The $\log(Q_T)$ versus log(frequency) curves of Figure 4 show Q_T to be roughly the same for similar fits at Mammoth and Monticello. Though it is generally agreed that Q_T is higher in the eastern U.S. the results here may be explained by the short lapse times of the codas used to estimate Q_T. These lapse times are much shorter than are usually reported in the literature for Q_T from coda estimates.

Table 5. Log(g) as Estimated from the S-wave Spectrum and Coda Source Function.

	Monticello		Mammoth	
	log g	freq	log g	freq
m = 0.5				
I	-2.50	3-72	-1.90	3-72
II	-2.53	3-72	-1.78	3-72
III	-3.27	3-72	-2.36	3-24
m = 1.0				
I	-3.03	3-72	-2.46	3-72
II	-2.96	3-72	-2.26	3-48
III	-3.50	3-72	-2.77	3-24

This implies that the results here represent the near surface Q_T and that it is approximately the same for both regions.

The results presented in Table 3 are the a and b values obtained by best fit of $Q_T = af^b$ to the log[Q_T] versus log[frequency] curves of Figure 5. The values of b in Table 3 range from 0.6 to 1.8, with an average of 1.3. The usual value of b reported in the literature is around 0.5 (0.6, Fedotov and Boldyrev, 1969; 0.5, Rautian and Khalturin, 1978; 0.5, Tsujiura, 1978; 0.6, Aki, 1980). However, the higher values obtained here are not unique. Rovelli (1982) has reported a b value of 1.1 for Friuli, Italy. This value was determined by coda analysis. Aki suggests that the b value, 0.8, which he determined for northeast Kanto, Japan, is the result of increased scattering by a higher density of scatterers associated with the higher level of tectonic activity in this part of the region. Aki and Chouet (1975) report on Q versus frequency at Stone Canyon, California. A b value of 0.9 is obtained by best fit of $Q_T = af^b$ to the reported results. Stone Canyon, Mammoth Lakes, and Friuli are all highly active regions.

The b value results at Monticello may be considered anomalous for this region of relative tectonic stability. However, in addition to including the reservoirs as a substantial part of the region sampled the codas and hypocentral distances at Monticello are much shorter than are usually reported in the literature for other studies. Consequently the near surface scatterers are favored in sampling. Andrews (1982) has suggested that near surface scattering is the dominant scattering effect for the similar situation at Mammoth Lakes.

Table 4 gives the results of fitting equation (5) to the values of Q_T as a function of frequency. Q_i and g are considered constants with frequency in making this fit. In (5), the two terms on the right hand side will be equal for a frequency F given by

$$F = gBQ_i/(2\pi) \tag{14}$$

Table 6 gives the values of F found using the results presented in Table 4. For frequencies less than F, scattering dominates the attenuation, while for frequencies greater than F, inelastic attenuation will dominate. From Table 6, scattering dominates attenuation over most of the frequency range in most cases, especially at Monticello. From (5), if scattering dominates then Q_T should be proportional to frequency, in approximate agreement with the results presented in Table 3. The proportionality of Q_T with frequency is also seen in Figure 4.

There is, however, another issue raised by Table 6. Dainty and Toksoz (1981) propose that if the frequency $f < F$, multiple scattering is a possibility. Obviously this situation holds for most of the codas considered here, from Table 6. Dainty and Toksoz point out that this seems to be true for many cases cited in the literature. However, Dainty and Toksoz's condition is only a necessary, not a sufficient, condition for multiple scattering. Since it will take some time after the origin time for multiple scattering to become important, a condition from Sato (1977) that multiple scattering will become important for times $t > T$, where $T = 1/(gB)$, is appropriate; if $f < F$ and $t > T$, strong scattering is probably occurring. T is also

Table 6. $F = gBQ_i/(2\pi)$, $T = 1/(gB)$. * is F tending to infinity.

	Monticello			Mammoth		
	F (Hz)	T (sec)	Freq. Range (Hz)	F (Hz)	T (sec)	Freq. Range (Hz)
m = 0.5						
I	60	2.4	3-72	30	5.4	3-72
II	50	2.9	3-72	15	8	3-72
III	*	4.4	3-72	20	9	3-24
m = 1.0						
I	*	1.4	3-72	40	4.2	3-72
II	4	24	3-72	100	4.2	3-48
III	*	9	3-72	5	19	3-24

given in Table 6, and in general indicates that multiple scattering might be occurring in some of the codas, remembering that I indicates $t \leq 5$ sec; II, $t \leq 9$ sec; III, $t > 9$ sec; but in practice t is less 30 sec for all codas in III. More generally, both F and T in Table 6 are of the same order of magnitude as the frequencies and times considered in this study, suggesting that the situation may be transitional between weak and strong scattering. A similar conclusion was reached by Dainty and Toksöz (1981). Theoretical formulations to handle this problem (e.g., Gao et al., 1983) do not appear to conserve energy and do not appear to agree with the strong scattering diffusion formalism of Dainty and Toksöz (1977).

The log[g] values obtained from the coda source spectrum are presented in Table 5. In general they are about an order of magnitude to two orders of magnitude smaller than the log[g] values obtained from Dainty's equation. Andrews (1982) has found the source spectral amplitude from the coda to be ten times larger than the spectral amplitude measured from the S-wave. As the value of g here is proportional to the quotient (coda source spectrum/S wave source spectrum), a small g value means our results do not agree with those of Andrews'. However, they are in agreement with the theoretical results reported in the second part of this report.

Throughout this report, the question of whether the scattering is two-dimensional ($m = 0.5$, cylindrical spreading, surface waves) or three-dimensional ($m = 1.0$, spherical spreading, body waves) has been left open. As discussed in the section on Analysis, it was not possible to determine the correct value of m from the data. Aki

(1980) has suggested on various grounds that the three-dimensional case is probably appropriate, but he dealt with much longer codas. Most of the more important results presented here seem to hold for both cases. It must be conceded, however, that the change in coda decay as time increases could be due to a change in Q_T reflecting the change from surface waves to body waves.

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A P P E N D I X I I

HIGH FREQUENCY ACOUSTIC BACKSCATTERING AND SEISMIC ATTENUATION

By Anton M. Dainty

Introduction

The phenomenon of scattering of seismic waves has attracted increasing attention in recent years. Backscattering of seismic waves has been used to explain terrestrial codas (Aki and Chouet, 1975; Dainty and Toksoz, 1981) and part or all of the attenuation suffered by seismic waves at frequencies of 1 Hz and greater (Aki, 1980; Dainty, 1981; Kikuchi, 1981; Wu, 1982). To analyse observed data on attenuation, coda amplitudes and other manifestations of scattering, a model of the earth as a scattering medium is required together with a theory to derive quantitative results from this model. Two general types of model have been used. One is the "randomly distributed specific scatterers" model. This model assumes scatterers of a specific type, for example spheres, randomly distributed throughout an otherwise homogeneous region. Examples of this type of model are the sphere model of Dainty (1981) used to explain the frequency dependence of the attenuation parameter Q at frequencies above 1 Hz, and the crack model of Kikuchi (1981), also used to analyse Q. A second type of model is the "random medium" model, in which the earth is considered to have average properties that do not vary from place to place, but with random fluctuations of the material properties. Examples are the use of an acoustic random medium by Aki (1980) and Wu (1982) to analyse Q above 1 Hz.

In this paper I shall examine the acoustic random medium model using the results of Chernov (1960) in the limit of high frequency backscattering. This problem was chosen because investigators using this model have generally concluded that this is the practical case in

the earth for frequencies of 1 Hz and higher. By analysing the high frequency limit, insight may be gained as to what type of information about the medium may be deduced from high frequency data. An additional benefit is an understanding of the relationship between the two types of models that have been used to examine this problem.

Random Medium Models

The simplest type of random medium used in seismology is the acoustic random medium discussed by Chernov (1960) (a summary of some useful results is given in Aki and Richards, 1980). The medium is considered to have fluctuations of velocity and density about mean values; for simplicity I will consider only the velocity fluctuations. Let the acoustic velocity C be given by

$$C = C_0(1-\mu(\vec{x})) \tag{1}$$

where C_0 is a constant, and $\mu(\vec{x})$, the slowness perturbation, is a random function of position $\vec{x}$ and has an average value of zero. The autocorrelation $N(\vec{r})$ of the slowness perturbation μ is then

$$N(\vec{r}) = \langle\mu(\vec{x})\mu(\vec{x}+\vec{r})\rangle/\langle\mu^2\rangle \tag{2}$$

The vector $\vec{r}$ is the lag, and the symbol $\langle\rangle$ means averaging over all $\vec{x}$. If the fluctuation is isotropic, then

$$N(\vec{r}) = N(r) \tag{3}$$

i.e., the autocorrelation depends only on the magnitude of the lag and not its direction.

Chernov (1960) finds the amplitude of singly (weakly) scattered waves for the case of a plane wave travelling through an acoustic medium of velocity C_0 without velocity fluctuations, except for a volume V containing velocity fluctuations as described above. If the incident (pressure) wave is given by

$$p_I = A \cdot \exp[2\pi fi(t-x/C_0)] \tag{4}$$

where f is the frequency, then the amplitude of the scattered wave at position $\vec{R}$ from the center of V in the limit of $\vec{R}$ large compared with the linear dimension of V is given by

$$|p_S|^2 = \frac{4\pi^2 V A^2 f^3 \langle\mu^2\rangle}{C_0^3 R^2 \sin(\theta/2)} \cdot \int_0^\infty N(r) \cdot \sin[(4\pi fr \cdot \sin(\theta/2))/C_0] \cdot r\,dr \tag{5}$$

(equation 45, p. 51, Chernov, 1960). The scattering angle θ is the angle between $\vec{R}$ and the direction of propagation; I shall define "back-scattering" as the case $\theta > \pi/2$. Equation (5) was first derived, in a slightly different form, by Pekeris (1947). This paper is concerned with the evaluation of (5) in the limit of large f and $\theta > \pi/2$.

Before proceeding, it should be noted that the acoustic medium described above is obviously too simple to describe the earth, which is

an elastic medium presumably containing fluctuations of both elastic constants and density, and through which two types of waves, compressional and shear, can propagate. Knopoff and Hudson (1967) have demonstrated that converted waves (scattered compressional waves from incident shear waves, and vice versa) are not important at high frequencies. Haddon and Cleary (1974) examined the problem of scattering from an inhomogeneous elastic medium; some of their (unpublished) results are discussed in Aki and Richards (1980). These results indicate that at least some of the equivalent formulas to equation (5) are of a similar form. Accordingly, equation (5) may be considered typical of integrals that appear in this type of problem.

Evaluation of the Backscattered Intensity at High Frequency

To evaluate (5), we need to find the integral

$$\int_0^{\infty} N(r) \cdot \sin[(4\pi fr \cdot \sin(\theta/2))/C_o] \cdot rdr = F(\infty) - F(0) \tag{6}$$

where F is the indefinite integral. However, if there is a scale length associated with the fluctuations, N(r) will generally decline rapidly for r larger than a. Accordingly, I will assume that $F(\infty) \to 0$ in (6); this will certainly be true of $N(r) \to \exp[-r/a]$ for large r. Then

$$\int_0^{\infty} N(r) \cdot \sin[(4\pi fr \cdot \sin(\theta/2))/C_o] \cdot rdr \cong -F(0) \tag{7}$$

To approximate F(0) in the case of f large and $\theta > \pi/2$, note that for this case the sine term in (7) varies much more rapidly than N(r), assuming N is a smooth function. Accordingly, I will expand N(r) in a Taylor series near r = 0. First, however, define

$$u = [4\pi fr \cdot \sin(\theta/2)]/C_o \tag{8}$$

and $$K = [4\pi f \cdot \sin(\theta/2)]/C_o \tag{9}$$

Then (7) becomes

$$\frac{1}{K^2} \int_0^{\infty} N(\tfrac{u}{K}) \cdot \sin(u) \cdot udu \cong -F(0) \tag{10}$$

Expand $N(\frac{u}{K})$ as, remembering that $u/K = r$,

$$N(\frac{u}{K}) = N(0) + \frac{u}{K}\,(\frac{dN}{dr})_{r=0} + \frac{1}{2}(\frac{u}{K})^2(\frac{d^2N}{dr^2})_{r=0} + \cdots \tag{11}$$

Then

$$F(u) \cong \frac{1}{K^2}\,[N(0)\int u\,.\,\sin(u)\,.\,du + \frac{1}{K}(\frac{dN}{dr})_{r=0}\int u^2\sin(u)\,.\,du$$

$$+ \frac{1}{2K^2}\,(\frac{d^2N}{dr^2})_{r=0}\int u^3\sin(u)\,.\,du + \ldots.] \tag{12}$$

To evaluate (12), use the tabulated integrals (Carmichael and Smith, 1962):

$$\int u\,.\,\sin(u)\,.\,du = \sin(u) - u\,.\,\cos(u) \tag{13}$$

$$\int u^m\sin(u)\,.\,du = -u^m\cos(u) + m\int u^{m-1}\cos(u)\,.\,du \tag{14}$$

$$\int u^m\cos(u)\,.\,du = u^m\sin(u) - m\int u^{m-1}\sin(u)\,.\,du \tag{15}$$

$$\int u\,.\,\cos(u)\,.\,du = \cos(u) + u\,.\,\sin(u) \tag{16}$$

Using (14) and (16),

$$\int u^2\sin(u)\,.\,du = 2u\,.\,\sin(u) + (2 - u^2)\cos(u) \tag{17}$$

Using (13), (14), (15), and (16)

$$\int u^m \sin(u) \,.\, du = \cos(u) \,.\, \left(-u^m + \sum_{n=1}^{\frac{m-1}{2}} (-1)^{n+1} m(m-1)\ldots(m-2n+1)u^{m-2n}\right) \quad \text{(m odd)} \qquad (18)$$

$$+ \sin(u)\left(mu^{m-1} + \sum_{n=1}^{\frac{m-1}{2}} (-1)^{n} m(m-1)\ldots(m-2n)u^{m-2n-1}\right)$$

$$= \cos(u)\left(-u^m + \sum_{n=1}^{m/2} (-1)^{n+1} m(m-1)\ldots(m-2n+1)u^{m-2n}\right) \quad \text{(m even)} \qquad (19)$$

$$+ \sin(u)\left(mu^{m-1} + \sum_{n=1}^{m/2} (-1)^{n} m(m-1)\ldots(m-2n)u^{m-2n-1}\right)$$

From (13), (17), (18), and (19), as $u \to 0$,

$$\int u \,.\, \sin(u) \,.\, du \to 0 \qquad (20)$$

$$\int u^2 \sin(u) \,.\, du \to 2 \qquad (21)$$

$$\int u^m \sin(u) \,.\, du \to 0 \quad \text{(m odd)} \qquad (22)$$

$$\to (-1)^{m/2+1} m! \quad \text{(m even)} \qquad (23)$$

Substituting in (12) and letting $u \to 0$,

$$F(0) = \sum_{n=1}^{\infty} \frac{1}{K^{2n+1}} (-1)^{n+1} \frac{(2n)!}{(2n-1)!} \left(\frac{d^{2n-1}N}{dr^{2n-1}}\right)_{r=0} \qquad (24)$$

According to (24), under the conditions described the backscattered intensity depends only on the odd derivatives of N at r=0, and not on the even derivatives. Since this is a high frequency approximation, the first term in (24) will be the most important, i.e.,

$$F(0) \cong \frac{2}{K^3} \left(\frac{dN}{dr}\right)_{r=0} + O(1/K^5) \tag{25}$$

Substituting in (10) and (5)

$$|p_s|^2 = - \frac{VA^2\langle\mu^2\rangle}{8\pi R^2 \sin^4(\theta/2)} \left(\frac{dN}{dr}\right)_{r=0} + O(1/f^2) \tag{26}$$

This indicates that the backscattered intensity is independent of frequency, provided that the autocorrelation has a non-zero derivative at zero lag. Since the first derivative at zero lag must be negative ($N(r)$ has a maximum at $r=0$), $|p_s|^2$ will be positive.

To test equation (26) I will compare it to two specified cases of $N(r)$ for which explicit formulas for $|p_s|^2$ are available (Chernov, 1960). If

$$N(r) = \exp(-r/a) \tag{27}$$

then

$$|p_s|^2 = \frac{32\pi^3 VA^2 f^4 a^3 \langle\mu^2\rangle}{R^2(C_o^4 + 16\pi^2 f^2 a^2 \sin^2(\theta/2))^2} \tag{28}$$

$$\rightarrow \frac{VA^2\langle\mu^2\rangle}{8\pi R^2 a \, . \, \sin^4(\theta/2)} \qquad (f \rightarrow \infty, \; \theta \geq \pi/2) \tag{29}$$

The approximate solution given by (26) reproduces this result.

Another case for which an exact solution is available is

$$N(r) = \exp(-r^2/a^2) \tag{30}$$

Then

$$|p_s|^2 = \frac{4\pi^{7/2}VA^2f^4a^3\langle\mu^2\rangle}{R^2}\exp[-(4\pi^2f^2a^2\sin^2(\theta/2))/C_o] \tag{31}$$

$$\rightarrow 0 \qquad (f \rightarrow \infty,\ \theta \geq \pi/2) \tag{32}$$

Since $N(r)$ has no non-zero derivatives of odd order for $r=0$, this is also in accord with (26).

Discussion

The limit of validity of (26) can be obtained from (7), since we require that the sine terms in (7) vary much more rapidly than N(r). If a is a scale length describing the range of r over which N(r) varies appreciably, we must have

$$(4\pi fa \,.\, \sin(\theta/2))/C_o >> 2\pi \tag{33}$$

or $$(2fa \,.\, \sin(\theta/2))/C_o >> 1 \tag{34}$$

Note that (34) requires that the scattering angle θ cannot be small; in fact,

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$$A(x) = A(0)\exp[-\pi fx/(QC_o)] \tag{36}$$

The total Q represents the effects of energy loss from the primary wave by the combined mechanisms of loss of seismic energy to heat (Q_i) and loss due to backscattering (Q_s) according to the relation (Warren, 1972):

$$1/Q = 1/Q_i + 1/Q_s \tag{37}$$

Wu (1982) derives an expression for Q_s, with the proper choice of V, as

$$1/Q_s = \frac{C_o R^2}{2\pi f A^2 V} \int_0^{2\pi} d\phi \int_{\pi/2}^{\pi} d\theta \left|p_s\right|^2 \sin(\theta) \tag{38}$$

where ϕ is an azimuthal angle about the direction of propagation of the primary wave and the integral over scattering angle θ is taken over all backscattering angles.

Substituting for $\left|p_s\right|^2$ from (26) and performing the integral over ϕ, in the limit of high frequency,

$$1/Q_s = -\frac{C_o\langle\mu^2\rangle}{8\pi f}\left(\frac{dN}{dr}\right)_{r=0} \int_{\pi/2}^{\pi} \frac{\sin(\theta)d\theta}{\sin^4(\theta/2)} \tag{39}$$

This may be evaluated using the trigonometric identity

$$2\sin^2(\theta/2) = 1 - \cos(\theta) \tag{40}$$

as

$$1/Q_s = -\frac{C_o\langle\mu^2\rangle}{4\pi f}\left(\frac{dN}{dr}\right)_{r=0} \tag{41}$$

Comparing this with an expression from Dainty (1981),

$$1/Q_s = gC_o/(2\pi f) \tag{42}$$

then

$$g = -\frac{\langle\mu^2\rangle}{2}\left(\frac{dN}{dr}\right)_{r=o} \tag{43}$$

This indicates that g, the turbidity, is independent of frequency and depends on the product of the mean square slowness fluctuation and the derivative of the autocorrelation of the slowness fluctuation. The lack of dependence of g on frequency is the same result obtained by Dainty (1981) for a set of randomly distributed spheres in geometric scatter; thus we may say that the approximation explored in this paper is the analog of geometric scattering from obstacles. Indeed, observationally we cannot distinguish between these cases, nor can we distinguish between different autocorrelation functions N(r) except insofar as their derivatives at r=0 differ.

In (38), the integral over the scattering angle was taken over all backscattered angles. Sato (1982a) has used a different approach (the mean wave method) from that used by Chernov (1960) and Wu (1982). His work suggests that instead of (38),

$$1/Q_s = \frac{C_o R^2}{2\pi f A^2 V} \int_0^{2\pi} d\phi \left[\int_{\theta_c}^{\pi} d\theta \left|p_s\right|^2 \sin(\theta) + \int_0^{\theta_c} d\theta \left|p_s\right|^2 \sin^4(\theta/2)\sin(\theta) \right] \quad (44)$$

$$\sin(\theta_c/2) = 1/4, \quad \theta_c \cong 30° \quad (45)$$

Comparing (44) and (38), Sato (1982a) indicates that all energy scattered at angles greater than θ_c should be considered lost from the primary wave (the first integral in the brackets) and a fraction of the more forward scattered energy should also be excluded (the second integral). Wu (1982) excluded only the backscattered energy. To evaluate (44) I shall use (26); while (26) is not valid near $\theta=0$, the presence of the weighting factor $[\sin^4(\theta/2)\sin(\theta)]$ in (44) ensures that the integral is small in this region. Then

$$1/Q_s \cong -\frac{7}{2\pi} \frac{C_o\langle\mu^2\rangle}{f} \left(\frac{dN}{dr}\right)_{r=0} \quad (46)$$

from (42), this gives

$$g \cong -7\langle\mu^2\rangle \left(\frac{dN}{dr}\right)_{r=0} \quad (47)$$

The value in (47) is larger than that in (43) because a greater part of the scattered energy was considered lost from the primary wave.

The results in (43) and/or (47) may be compared with another parameter, the backscattering turbidity

$$g_\pi = \frac{R^2}{AV} \left| p_s(\pi) \right|^2 \tag{48}$$

$$= \frac{\langle \mu^2 \rangle}{8\pi} \left(\frac{dN}{dr}\right)_{r=0} \tag{49}$$

This parameter controls the amplitude level of coda waves relative to the direct wave, after correction for geometrical spreading and attenuation (Aki and Chouet, 1975). Aki (1980) suggested that $g_\pi \sim g$; Sato (1982b) presented a more detailed calculation. According to the formulas presented here,

$$g_\pi/g = 1/(4\pi) \cong 10\% \tag{50}$$

if expression (43) is used for g,

$$g_\pi/g \cong 1/(56\pi) \cong 1\% \tag{51}$$

if (47) is used for g.

A final, important question has to do with the nature of the slowness fluctuation $\mu(\vec{x})$ that will lead to an autocorrelation N(r) that has a non-zero first derivative at r=0. Chernov (1960) asserts that the first derivative of N(r) must tend to zero for $r \to 0$ if $\mu(\vec{x})$ is continuous, although a proof is only given for the one-dimensional case. This is a special case of a result due to Taylor (1920), who proved (for the one-dimensional case) that the autocorrelation of a continuous function has only even-ordered derivatives that are non-zero at zero

lag. Pekeris (1942) comments further on Taylor's result. However, K. Aki (personal communication) has pointed out that the (one-dimensional) function

$$\mu(x) = \int_{-\infty}^{\infty} w(\tau)h(x-\tau)d\tau \tag{52}$$

$$h(x) = \exp[-x/a] \qquad x \geq 0 \tag{53}$$

$$= 0 \qquad x < 0$$

has the autocorrelation given by (27) if w(x) is any function with a power spectrum that is constant with frequency. If w(x) is chosen to be "white noise", it appears that (52) is continuous, even though it has an autocorrelation which has a non-zero first derivative at zero lag. However, the first derivative of (52) is

$$\mu'(x) = \int_{-\infty}^{\infty} w(\tau)h'(x-\tau)d\tau$$

$$= \int_{-\infty}^{\infty} w(\tau)[\delta(x-\tau) - (1/a)h(x-\tau)]d\tau$$

$$= w(x) - (1/a)\mu(x) \tag{54}$$

This is a discontinuous function, since w(x) is discontinuous. Thus the function $\mu(x)$ is "continuous" but is not "smooth". This property of $\mu(x)$ is perhaps the reason that Taylor's (1920) and Chernov's (1960) proofs fail. The lack of smoothness may explain why the results

obtained here are equivalent to geometrical scattering from obstacles, since in this case the scattering is controlled by the discontinuity which is the boundary of the obstacle.

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HIGH FREQUENCY ACOUSTIC BACKSCATTERING

AND SEISMIC ATTENUATION

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Abstract

An expression for the backscattered intensity of acoustic waves singly scattered from a region containing fluctuations of the acoustic velocity has been derived for high frequencies by expanding the autocorrelation of the slowness fluctuations in a Taylor series about zero lag. The resulting expression indicates that the backscattered intensity is independent of frequency and directly proportional to the first derivative of the autocorrelation at zero lag; the next higher term is proportional to the reciprocal of the square of the frequency and directly proportional to the third derivative of the autocorrelation at zero lag. Contributions from terms of the Taylor series involving even numbered derivatives of the autocorrelation are zero. Since for the autocorrelation of a smooth function only the even numbered derivatives are non-zero at zero lag, this result demonstrates that backscattering at high frequencies can only occur from discontinuities of velocity or its derivatives as opposed to fluctuations in which the velocities are smooth. If the backscattered intensity is independent of frequency, the contribution of backscattering to the attenuation parameter Q is proportional to frequency. Such behavior may have been observed for seismic waves of frequencies greater than 1 Hz, suggesting that scattering from discontinuities is an important part of the attenuation of such waves.

Introduction

The phenomenon of scattering of seismic waves has attracted increasing attention in recent years. Backscattering of seismic waves has been used to explain terrestrial codas (Aki and Chouet, 1975; Dainty and Toksoz, 1981) and part or all of the attenuation suffered by seismic waves at frequencies of 1 Hz and greater (Aki, 1980; Dainty, 1981; Kikuchi, 1981; Wu, 1982). To analyse observed data on attenuation, coda amplitudes and other manifestations of scattering, a model of the earth as a scattering medium is required together with a theory to derive quantitative results from this model. Two general types of model have been used. One is the "randomly distributed specific scatterers" model. This model assumes scatterers of a specific type, for example spheres, randomly distributed throughout an otherwise homogeneous region. Examples of this type of model are the sphere model of Dainty (1981) used to explain the frequency dependence of the attenuation parameter Q at frequencies above 1 Hz, and the crack model of Kikuchi (1981), also used to analyse Q. A second type of model is the "random medium" model, in which the earth is considered to have average properties that do not vary from place to place, but with random fluctuations of the material properties. Examples are the use of an acoustic random medium by Aki (1980) and Wu (1982) to analyse Q above 1 Hz.

In this paper I shall examine the acoustic random medium model using the results of Chernov (1960) in the limit of high frequency backscattering. This problem was chosen because investigators using this model have generally concluded that this is the practical case in

the earth for frequencies of 1 Hz and higher. By analysing the high frequency limit, insight may be gained as to what type of information about the medium may be deduced from high frequency data. An additional benefit is an understanding of the relationship between the two types of models that have been used to examine this problem.

Random Medium Models

The simplest type of random medium used in seismology is the acoustic random medium discussed by Chernov (1960) (a summary of some useful results is given in Aki and Richards, 1980). The medium is considered to have fluctuations of velocity and density about mean values; for simplicity I will consider only the velocity fluctuations. Let the acoustic velocity C be given by

$$C = C_0(1-\mu(\vec{x})) \tag{1}$$

where C_0 is a constant, and $\mu(\vec{x})$, the slowness perturbation, is a random function of position $\vec{x}$ and has an average value of zero. The autocorrelation $N(\vec{r})$ of the slowness perturbation μ is then

$$N(\vec{r}) = \langle\mu(\vec{x})\mu(\vec{x}+\vec{r})\rangle/\langle\mu^2\rangle \tag{2}$$

The vector $\vec{r}$ is the lag, and the symbol $\langle\rangle$ means averaging over all $\vec{x}$. If the fluctuation is isotropic, then

$$N(\vec{r}) = N(r) \tag{3}$$

i.e., the autocorrelation depends only on the magnitude of the lag and not its direction.

Chernov (1960) finds the amplitude of singly (weakly) scattered waves for the case of a plane wave travelling through an acoustic medium of velocity C_0 without velocity fluctuations, except for a volume V containing velocity fluctuations as described above. If the incident (pressure) wave is given by

$$p_I = A \cdot \exp[2\pi fi(t-x/C_0)] \quad . \tag{4}$$

where f is the frequency, then the amplitude of the scattered wave at position $\vec{R}$ from the center of V in the limit of $\vec{R}$ large compared with the linear dimension of V is given by

$$|p_s|^2 = \frac{4\pi^2 VA^2 f^3 \langle\mu^2\rangle}{C_0^3 R^2 \sin(\theta/2)} \cdot \int_0^\infty N(r) \cdot \sin[(4\pi fr \cdot \sin(\theta/2))/C_0] \cdot rdr \tag{5}$$

(equation 45, p. 51, Chernov, 1960). The scattering angle θ is the angle between $\vec{R}$ and the direction of propagation; I shall define "back-scattering" as the case $\theta > \pi/2$. Equation (5) was first derived, in a slightly different form, by Pekeris (1947). This paper is concerned with the evaluation of (5) in the limit of large f and $\theta > \pi/2$.

Before proceeding, it should be noted that the acoustic medium described above is obviously too simple to describe the earth, which is

an elastic medium presumably containing fluctuations of both elastic constants and density, and through which two types of waves, compressional and shear, can propagate. Knopoff and Hudson (1967) have demonstrated that converted waves (scattered compressional waves from incident shear waves, and vice versa) are not important at high frequencies. Haddon and Cleary (1974) examined the problem of scattering from an inhomogeneous elastic medium; some of their (unpublished) results are discussed in Aki and Richards (1980). These results indicate that at least some of the equivalent formulas to equation (5) are of a similar form. Accordingly, equation (5) may be considered typical of integrals that appear in this type of problem.

Evaluation of the Backscattered Intensity at High Frequency

To evaluate (5), we need to find the integral

$$\int_0^\infty N(r) \cdot \sin[(4\pi fr \cdot \sin(\theta/2))/C_0] \cdot rdr = F(\infty) - F(0) \tag{6}$$

where F is the indefinite integral. However, if there is a scale length associated with the fluctuations, N(r) will generally decline rapidly for r larger than a. Accordingly, I will assume that $F(\infty) \to 0$ in (6); this will certainly be true of $N(r) \to \exp[-r/a]$ for large r. Then

$$\int_0^\infty N(r) \cdot \sin[(4\pi fr \cdot \sin(\theta/2))/C_0] \cdot rdr \cong -F(0) \tag{7}$$

To approximate F(0) in the case of f large and $\theta > \pi/2$, note that for this case the sine term in (7) varies much more rapidly than N(r), assuming N is a smooth function. Accordingly, I will expand N(r) in a Taylor series near r = 0. First, however, define

$$u = [4\pi fr \cdot \sin(\theta/2)]/C_0 \tag{8}$$

and

$$K = [4\pi f \cdot \sin(\theta/2)]/C_0 \tag{9}$$

Then (7) becomes

$$\frac{1}{K^2} \int_0^\infty N(\frac{u}{K}) \cdot \sin(u) \cdot udu \cong -F(0) \tag{10}$$

Expand $N(\frac{u}{K})$ as, remembering that $u/K = r$,

$$N(\frac{u}{K}) = N(0) + \frac{u}{K}\left(\frac{dN}{dr}\right)_{r=0} + \frac{1}{2}\left(\frac{u}{K}\right)^2\left(\frac{d^2N}{dr^2}\right)_{r=0} + \ldots \tag{11}$$

Then

$$F(u) \cong \frac{1}{K^2}\Big[N(0)\int u \,.\, \sin(u) \,.\, du + \frac{1}{K}\left(\frac{dN}{dr}\right)_{r=0}\int u^2 \sin(u) \,.\, du$$

$$+ \frac{1}{2K^2}\left(\frac{d^2N}{dr^2}\right)_{r=0}\int u^3 \sin(u) \,.\, du + \ldots\Big] \tag{12}$$

To evaluate (12), use the tabulated integrals (Carmichael and Smith, 1962):

$$\int u \,.\, \sin(u) \,.\, du = \sin(u) - u \,.\, \cos(u) \tag{13}$$

$$\int u^m \sin(u) \,.\, du = -u^m \cos(u) + m\int u^{m-1}\cos(u) \,.\, du \tag{14}$$

$$\int u^m \cos(u) \,.\, du = u^m \sin(u) - m\int u^{m-1}\sin(u) \,.\, du \tag{15}$$

$$\int u \,.\, \cos(u) \,.\, du = \cos(u) + u \,.\, \sin(u) \tag{16}$$

Using (14) and (16),

$$\int u^2 \sin(u) \,.\, du = 2u \,.\, \sin(u) + (2 - u^2)\cos(u) \tag{17}$$

Using (13), (14), (15), and (16)

$$\int u^m \sin(u) \,.\, du = \cos(u) \,.\, \left(-u^m + \sum_{n=1}^{\frac{m-1}{2}} (-1)^{n+1} m(m-1)\ldots(m-2n+1)u^{m-2n}\right) \qquad \text{(m odd)} \quad (18)$$

$$+ \sin(u)\left(mu^{m-1} + \sum_{n=1}^{\frac{m-1}{2}} (-1)^n m(m-1)\ldots(m-2n)u^{m-2n-1}\right)$$

$$= \cos(u)\left(-u^m + \sum_{n=1}^{m/2} (-1)^{n+1} m(m-1)\ldots(m-2n+1)u^{m-2n}\right) \qquad \text{(m even)} \quad (19)$$

$$+ \sin(u)\left(mu^{m-1} + \sum_{n=1}^{m/2} (-1)^n m(m-1)\ldots(m-2n)u^{m-2n-1}\right)$$

From (13), (17), (18), and (19), as $u \to 0$,

$$\int u \,.\, \sin(u) \,.\, du \to 0 \qquad (20)$$

$$\int u^2 \sin(u) \,.\, du \to 2 \qquad (21)$$

$$\int u^m \sin(u) \,.\, du \to 0 \qquad \text{(m odd)} \qquad (22)$$

$$\to (-1)^{m/2+1} m! \qquad \text{(m even)} \qquad (23)$$

Substituting in (12) and letting $u \to 0$,

$$F(0) = \sum_{n=1}^{\infty} \frac{1}{K^{2n+1}} (-1)^{n+1} \frac{(2n)!}{(2n-1)!} \left(\frac{d^{2n-1}N}{dr^{2n-1}}\right)_{r=0} \qquad (24)$$

According to (24), under the conditions described the backscattered intensity depends only on the odd derivatives of N at r=0, and not on the even derivatives. Since this is a high frequency approximation, the first term in (24) will be the most important, i.e.,

$$F(0) \cong \frac{2}{K^3}\left(\frac{dN}{dr}\right)_{r=0} + O(1/K^5) \tag{25}$$

Substituting in (10) and (5)

$$|p_s|^2 = -\frac{VA^2\langle\mu^2\rangle}{8\pi R^2 \sin^4(\theta/2)}\left(\frac{dN}{dr}\right)_{r=0} + O(1/f^2) \tag{26}$$

This indicates that the backscattered intensity is independent of frequency, provided that the autocorrelation has a non-zero derivative at zero lag. Since the first derivative at zero lag must be negative ($N(r)$ has a maximum at $r=0$), $|p_s|^2$ will be positive.

To test equation (26) I will compare it to two specified cases of $N(r)$ for which explicit formulas for $|p_s|^2$ are available (Chernov, 1960). If

$$N(r) = \exp(-r/a) \tag{27}$$

then

$$|p_s|^2 = \frac{32\pi^3 VA^2 f^4 a^3 \langle\mu^2\rangle}{R^2(C_0{}^4 + 16\pi^2 f^2 a^2 \sin^2(\theta/2))^2} \tag{28}$$

$$\rightarrow \frac{VA^2\langle\mu^2\rangle}{8\pi R^2 a \,.\, \sin^4(\theta/2)} \qquad (f \rightarrow \infty,\ \theta \geq \pi/2) \tag{29}$$

The approximate solution given by (26) reproduces this result.

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Discussion

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Wu (1982) derives an expression for Q_s, with the proper choice of V, as

$$1/Q_s = \frac{C_o R^2}{2\pi f A^2 V} \int_0^{2\pi} d\phi \int_{\pi/2}^{\pi} d\theta \left|p_s\right|^2 \sin(\theta) \tag{38}$$

where ϕ is an azimuthal angle about the direction of propagation of the primary wave and the integral over scattering angle θ is taken over all backscattering angles.

Substituting for $\left|p_s\right|^2$ from (26) and performing the integral over ϕ, in the limit of high frequency,

$$1/Q_s = -\frac{C_o\langle\mu^2\rangle}{8\pi f} \left(\frac{dN}{dr}\right)_{r=0} \int_{\pi/2}^{\pi} \frac{\sin(\theta)d\theta}{\sin^4(\theta/2)} \tag{39}$$

This may be evaluated using the trigonometric identity

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as

$$1/Q_s = - \frac{C_0\langle\mu^2\rangle}{4\pi f} \left(\frac{dN}{dr}\right)_{r=0} \tag{41}$$

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This indicates that g, the turbidity, is independent of frequency and depends on the product of the mean square slowness fluctuation and the derivative of the autocorrelation of the slowness fluctuation. The lack of dependence of g on frequency is the same result obtained by Dainty (1981) for a set of randomly distributed spheres in geometric scatter; thus we may say that the approximation explored in this paper is the analog of geometric scattering from obstacles. Indeed, observationally we cannot distinguish between these cases, nor can we distinguish between different autocorrelation functions N(r) except insofar as their derivatives at r=0 differ.

In (38), the integral over the scattering angle was taken over all backscattered angles. Sato (1982a) has used a different approach (the mean wave method) from that used by Chernov (1960) and Wu (1982). His work suggests that instead of (38),

$$1/Q_s = \frac{C_o R^2}{2\pi f A^2 V} \int_0^{2\pi} d\phi \left[\int_{\theta_c}^{\pi} d\theta \left|p_s\right|^2 \sin(\theta) + \int_0^{\theta_c} d\theta \left|p_s\right|^2 \sin^4(\theta/2)\sin(\theta) \right] \quad (44)$$

$$\sin(\theta_c/2) = 1/4, \quad \theta_c \cong 30° \quad (45)$$

Comparing (44) and (38), Sato (1982a) indicates that all energy scattered at angles greater than θ_c should be considered lost from the primary wave (the first integral in the brackets) and a fraction of the more forward scattered energy should also be excluded (the second integral). Wu (1982) excluded only the backscattered energy. To evaluate (44) I shall use (26); while (26) is not valid near θ=0, the presence of the weighting factor [$\sin^4(\theta/2)\sin(\theta)$] in (44) ensures that the integral is small in this region. Then

$$1/Q_s \cong - \frac{7}{2\pi} \frac{C_o \langle\mu^2\rangle}{f} \left(\frac{dN}{dr}\right)_{r=0} \quad (46)$$

from (42), this gives

$$g \cong -7\langle\mu^2\rangle \left(\frac{dN}{dr}\right)_{r=0} \quad (47)$$

The value in (47) is larger than that in (43) because a greater part of the scattered energy was considered lost from the primary wave.

The results in (43) and/or (47) may be compared with another parameter, the backscattering turbidity

$$g_\pi = \frac{R^2}{AV} \left| p_s(\pi) \right|^2 \tag{48}$$

$$= \frac{\langle\mu^2\rangle}{8\pi} \left(\frac{dN}{dr}\right)_{r=0} \tag{49}$$

This parameter controls the amplitude level of coda waves relative to the direct wave, after correction for geometrical spreading and attenuation (Aki and Chouet, 1975). Aki (1980) suggested that $g_\pi \sim g$; Sato (1982b) presented a more detailed calculation. According to the formulas presented here,

$$g_\pi/g = 1/(4\pi) \cong 10\% \tag{50}$$

if expression (43) is used for g,

$$g_\pi/g \cong 1/(56\pi) \cong 1\% \tag{51}$$

if (47) is used for g.

A final, important question has to do with the nature of the slowness fluctuation $\mu(\vec{x})$ that will lead to an autocorrelation N(r) that has a non-zero first derivative at r=0. Chernov (1960) asserts that the first derivative of N(r) must tend to zero for $r \rightarrow 0$ if $\mu(\vec{x})$ is continuous, although a proof is only given for the one-dimensional case. This is a special case of a result due to Taylor (1920), who proved (for the one-dimensional case) that the autocorrelation of a continuous function has only even-ordered derivatives that are non-zero at zero

lag. Pekeris (1942) comments further on Taylor's result. However, K. Aki (personal communication) has pointed out that the (one-dimensional) function

$$\mu(x) = \int_{-\infty}^{\infty} w(\tau)h(x-\tau)d\tau \tag{52}$$

$$h(x) = \exp[-x/a] \qquad x \geq 0 \tag{53}$$

$$= 0 \qquad x < 0$$

has the autocorrelation given by (27) if w(x) is any function with a power spectrum that is constant with frequency. If w(x) is chosen to be "white noise", it appears that (52) is continuous, even though it has an autocorrelation which has a non-zero first derivative at zero lag. However, the first derivative of (52) is

$$\mu'(x) = \int_{-\infty}^{\infty} w(\tau)h'(x-\tau)d\tau$$

$$= \int_{-\infty}^{\infty} w(\tau)[\delta(x-\tau) - (1/a)h(x-\tau)]d\tau$$

$$= w(x) - (1/a)\mu(x) \tag{54}$$

This is a discontinuous function, since w(x) is discontinuous. Thus the function $\mu(x)$ is "continuous" but is not "smooth". This property of $\mu(x)$ is perhaps the reason that Taylor's (1920) and Chernov's (1960) proofs fail. The lack of smoothness may explain why the results

obtained here are equivalent to geometrical scattering from obstacles, since in this case the scattering is controlled by the discontinuity which is the boundary of the obstacle.

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